THIRD YEAR CIVIL

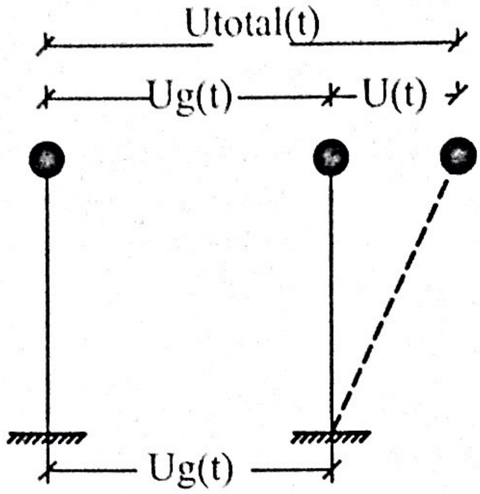
STRUCTURAL ANALYSIS

DYNAMICS

GROUND MOVEMENT

5

GROUND ACCELERATION



$$M\,U''total(t) + K\,U(t) = 0$$

$$M\,[Ug''(t) + U''(t)] + K\,U(t) = 0$$

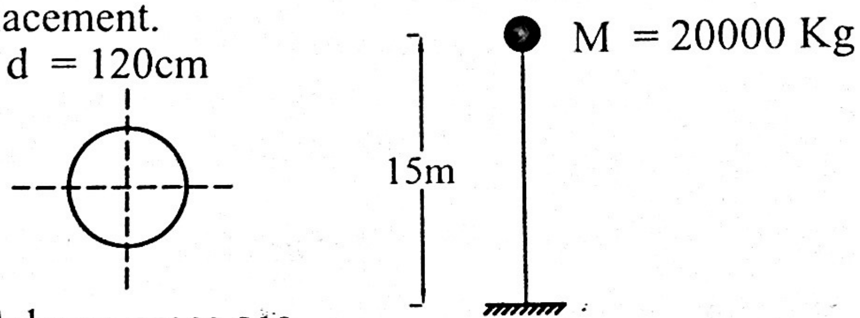
$$\boxed{M\,U''(t) + K\,U(t) = -M\,U''g(t)}$$

EXAMPLE

For the shown elevated tank if it is subjected to a horizontal acceleration of support of (2 Sin 16.88 t) m / sec^2 neglecting the mass of the column and the damping . Calculate

1) Stiffness
2) Natural frequency
3) Max. displacement.
4) Stress on Column Base due to static loading.
5) Stress on Column Base due to dynamic loading.
6) Time of max. displacement.

(E = 20 GPa)



Column cross sec.

$$M X''(t) + K X(t) = - K X(t)$$

$$M U''(t) + K U(t) = - M X''s(t)$$

$$1)\ K = \frac{3EI}{L^3} = \frac{(3)(2 \times 10^{10})(\pi \times 1.2^4 / 64)}{(15^3)} = 1.81 \times 10^6 \text{ N/m}$$

$$2)\ \omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{1.81 \times 10^6}{20000}} = 9.5 \text{ rad/sec}$$

$$20000\, U''(t) + 1.8 \times 10^6\, U(t) = - 40000 \text{ Sin } 16.88\, t$$

$$U(t) = \frac{P_0}{K} \frac{1}{1 - (\beta)^2} (\text{Sin } \Omega t - \beta \text{ Sin } \omega t) \text{ where } \beta = \frac{\Omega}{\omega}$$

$$\beta = (16.88 / 9.5) = 1.778$$

$$U(t) = \frac{40000}{1.81 \times 10^6} \frac{1}{1 - (1.778)^2} (\text{Sin } 16.88\, t - 1.778 \text{ Sin } 9.5\, t)$$

3) Max. displacement.

$$U\ max. = \frac{P_0}{K}\ \frac{1}{1-\beta^2} = \frac{40000}{1.81x10^6}\ \frac{1}{1-3.158} = -0.01\ m$$

4) <u>Stresses on Column base due to static loading :</u>

$$f = \frac{N}{A}$$

لتحويل ال Mass الى Weight

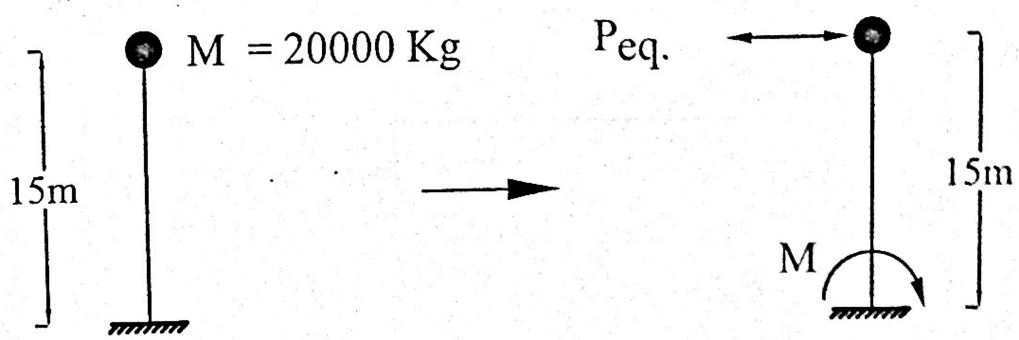
Weight (N) = Mass (Kg) x g (m / sec^2)

g = 9.81 m / sec^2 and we can take it 10 m / sec^2

W = 20000 (Kg) x 10 (m / sec^2) = 200000 N

$$f = \frac{W}{A} = \frac{200000}{\pi\,(1.2)^2/4} = 176839\ N/m^2$$

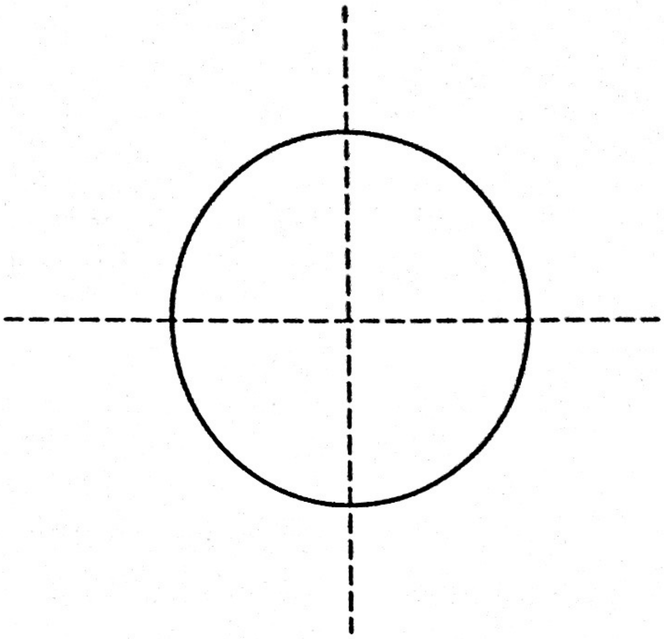
5) <u>Stresses on Column base due to dynamic loading :</u>

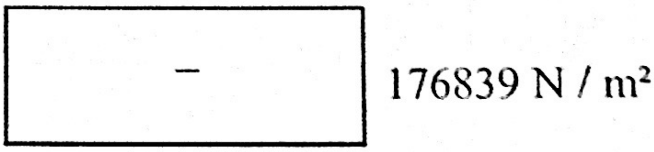


P eq. = K U max. = 1.81 x 10^6 x 0.01 = 18100 N

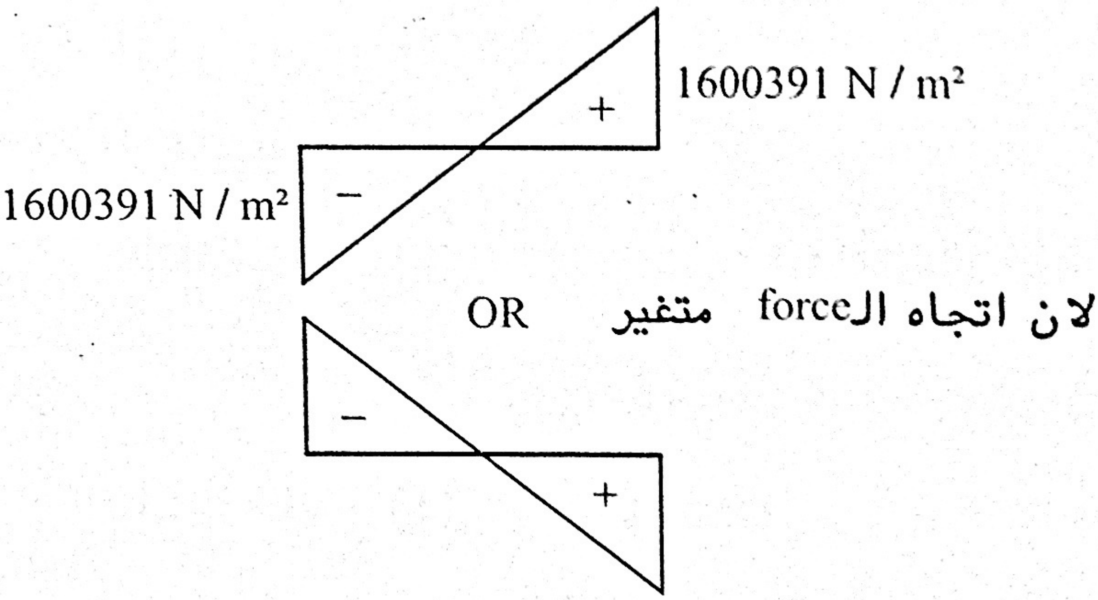
M = P eq. x H = 18100 x 15 = 271500 N. m

$$f\,(\,dynamic\,) = \frac{M}{I}\,Y = \frac{271500}{\pi\,(1.2)^4/64} \times 0.6 = 1600391\ N/m^2$$





Static stress distribution



dynamic stress distribution

Max. total Comp. Stress = - 1600391 - 176839 = - 1777230 N / m²

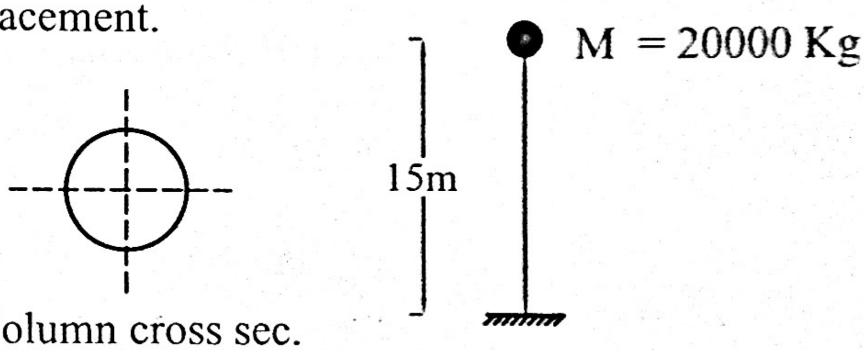
Max. total tens. Stress = 1600391 - 176839 = 1423452 N / m²

EXAMPLE (2)

For the shown elevated tank if it is subjected to a vertical acceleration of support of (2 Sin 90 t) m / sec^2 neglecting the mass of the column and the damping . Calculate

1) Stiffness
2) Natural frequency
3) Max. displacement.
4) Stress on Column Base due to static loading.
5) Stress on Column Base due to dynamic loading.
6) Time of max. displacement.

(E = 20 GPa)



Column cross sec.

$$M X''(t) + K X(t) = - K X(t)$$

$$M U''(t) + K U(t) = - M X''_s(t)$$

$$1)\ K = \frac{E A}{L} = \frac{(2 \times 10^{10})(\pi \times 1.2^2 / 4)}{(15)} = 1.508 \times 10^9 \ N/m$$

$$2)\ \omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{1.508 \times 10^9}{20000}} = 274.59 \ rad/sec$$

$$20000\, U''(t) + 1.508 \times 10^9\, U(t) = - 40000 \ Sin\, 90\, t$$

$$U(t) = \frac{P_0}{K} \frac{1}{1-(\beta)^2} (Sin\, \Omega t - \beta\, Sin\, \omega t) \quad where\ \beta = \frac{\Omega}{\omega}$$

$$\beta = (90 / 274.59) = 0.327$$

$$U(t) = \frac{40000}{1.508 \times 10^9} \frac{1}{1-(0.327)^2} (Sin\, 90\, t - 0.327\, Sin\, 274.59\, t)$$

3) Max. displacement.

$$U\ max. = \frac{P_0}{K} \quad \frac{1}{1 - \beta^2} = \frac{40000}{1.508x10^9} \quad \frac{1}{1 - 0.109} = 0.0000298\ m$$

4) <u>Stresses on Column base due to static loading :</u>

$$f = \frac{N}{A}$$

لتحويل الـMass الى Weight

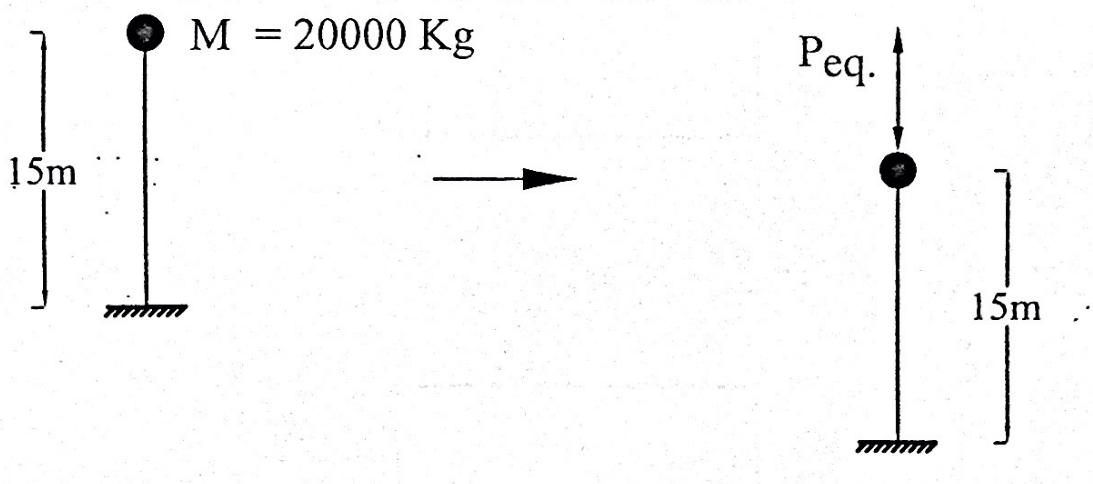
Weight (N) = Mass (Kg) x g (m / sec²)

g = 9.81 m / sec² and we can take it 10 m / sec²

W = 20000 (Kg) x 10 (m / sec²) = 200000 N

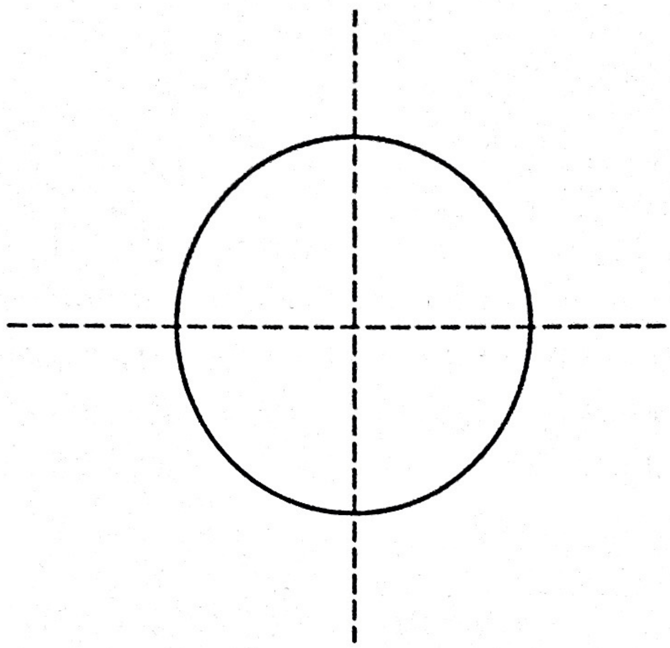
$$f = \frac{W}{A} = \frac{200000}{\pi\,(1.2)^2/4} = 176839\ N/m^2$$

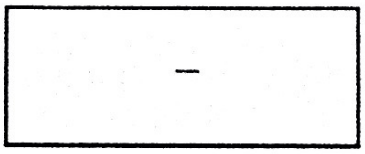
5) <u>Stresses on Column base due to dynamic loading :</u>



$$P\ eq. = K\ U\ max. = 1.508 \times 10^9 \times 0.0000298 = 44893.4\ N$$

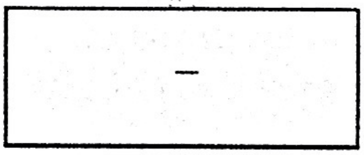
$$f = \frac{N}{A} = \frac{44893.4}{\pi\,(1.2)^2/4} = 39694\ N/m^2$$





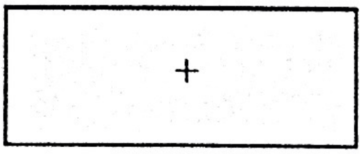
176839 N / m²

Static stress distribution



39694 N / m²

لان اتجاه ال force متغير   OR



39694 N / m²

dynamic stress distribution

Max. total Comp. Stress = - 39694 - 176839 = - 216533 N / m²

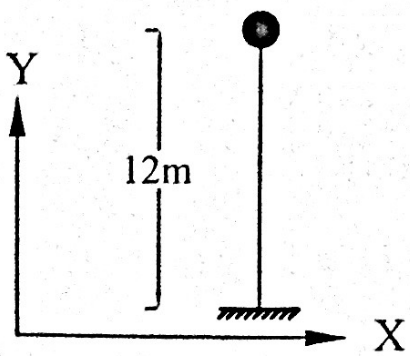
Max. total tens. Stress = 39694 - 176839 = -137145 N / m² (no tens.)

SHEET (5)

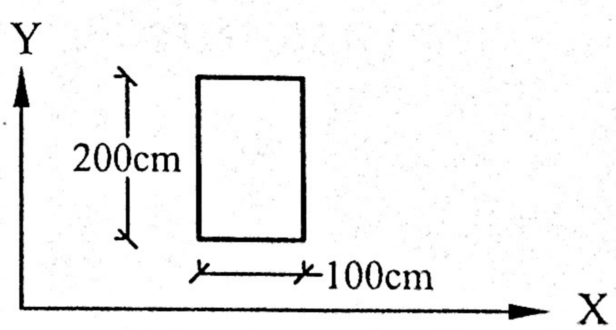
A mass of 100,000 kg is supported on the shown column. The column cross-section is rectangular 100 x 200 cms and young's modulus for the column material is 2×10^{10} N/m². The column mass is to be neglected. An earthquake produces harmonic loads on the lumped mass in three directions at the same time expressed as follows.assume damping is equal to 0.10 .

In X direction : F (t) = (78480) Sin (10 t)
In Y direction : F (t) = (147150) Sin (10 t)
In Z direction : F (t) = (49050) Sin (10 t)

1) Write down the differential equation of motion of the mass in each direction.
2) Determine the maximum exepected dynamic displaement of the mass in each direction separately.
3) Determine the stresses in column's section due to static loading.
4) Determine the maximum stresses ic column's section due to the dynamic vibration in each direction as a separate case of loading.
5) Determine the maximum combined total stresses due to dynamic loading in all directions simaltanously.
6) Determine the maximum total stresses due to both static and dynamic loading.
7) What is the minimum weight that will produce resonance state.



Tower elevation



Column section

EXAMPLE (1)

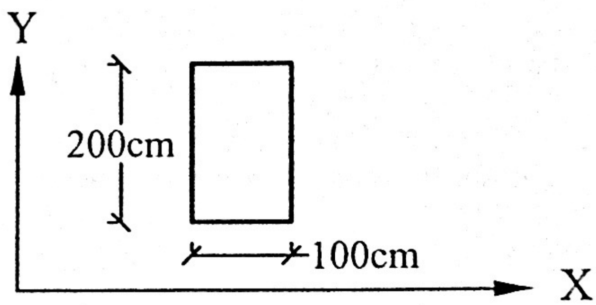
$M = 100,000$ kg , Weight $= 100,000 \times 9.81 = 981,000$ N

$E = 2 \times 10^{10}$ N/m² , $I_x = (1)(2)^3 / 12 = 0.667$ m⁴

$I_y = (2)(1)^3 / 12 = 0.167$ m⁴ , $A = 1 \times 2 = 2$ m²

$\xi = 0.10$

(1)



For (X) direction.

$$M\,U'' + C\,U' + K\,U = P_o \sin \Omega t$$

$$K = 3\,E\,I_y / L^3 = 3 \times 2 \times 10^{10} \times 0.167 / 12^3 = 5798611 \text{ N / m}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{5798611 \text{ N / m}}{100,000 \text{ kg}}} = 7.61 \text{ rad / sec}$$

$$C = \xi\, C_{Cr} = 0.10 \times 2 \times 7.61 \times 100,000 = 152200 \text{ N . sec / m}$$

$$(100,000)\,U'' + (152200)\,U' + (5798611)\,U = (78480)\,\sin(10)\,t$$

For (Y) direction.

$$M\,U'' + C\,U' + K\,U = P_o \sin \Omega t$$

$$K = 3\,E\,I_X / L^3 = 3 \times 2 \times 10^{10} \times 0.667 / 12^3 = 23159722 \text{ N / m}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{23159722 \text{ N / m}}{100,000 \text{ kg}}} = 15.21 \text{ rad / sec}$$

$$C = \xi\, C_{Cr} = 0.10 \times 2 \times 15.21 \times 100,000 = 304200 \text{ N . sec / m}$$

$$(100,000)\,U'' + (304200)\,U' + (23159722)\,U = (147150)\,\sin(10)\,t$$

For (Y) direction.

$$M\,U'' + C\,U' + K\,U = P_0 \,\text{Sin}\; \Omega\, t$$

$$K = E\,A\,/\,L = 2 \times 10 \;\times 2^{10} /\, 12 = 3.33 \times 10^{9}\; N\,/\,m$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{3.33 \times 10^{9}\; N\,/m}{100{,}000\; kg}} = 182.61\; rad\,/\,sec$$

$$C = \xi\, C_{Cr} = 0.10 \times 2 \times 182.61 \times 100{,}000 = 3652200\; N.\,sec\,/\,m$$

$$(\,100{,}000\,)\,U'' + (\,3652200\,)\,U' + (\,3.33 \times 10^{9})\,U = (\,49050\,)\;\text{Sin}\,(\,10\,)\,t$$

$$(\,2\,)\quad \Delta\,max. = \frac{P_0}{K}\;\frac{1}{\sqrt{(1-\beta^2)^2 + (\,2\,\zeta\,\beta)^2}}$$

For (X) direction.

$P_0 = 78480\; N$, $\Omega = 10$, $\omega = 7.61$, $K = 5798611\; N\,/\,m$

$$\beta = \frac{\Omega}{\omega} = \frac{10}{7.61} = 1.314$$

$$\Delta\,max. = \frac{P_0}{K}\;\frac{1}{\sqrt{(1-\beta^2)^2 + (\,2\,\zeta\,\beta)^2}}$$

$$= \frac{78480}{5798611}\;\frac{1}{\sqrt{(1-1.314^2)^2 + (\,2 \times 0.1 \times 1.314)^2}} = 0.0179\; m$$

For (Y) direction.

$P_0 = 147150\ N \quad , \Omega = 10 \quad , \omega = 15.21 \quad , K = 23159722\ N/m$

$$\beta = \frac{\Omega}{\omega} = \frac{10}{15.21} = 0.657$$

$$\Delta\ max. = \frac{P_0}{K} \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}$$

$$= \frac{147150}{23159722} \frac{1}{\sqrt{(1-0.657^2)^2 + (2\ x0.1x0.657)^2}} = 0.0111\ m$$

For (Z) direction.

$P_0 = 49050\ N \quad , \Omega = 10 \quad , \omega = 182.61 \quad , K = 3.33\ x\ 10^9\ N/m$

$$\beta = \frac{\Omega}{\omega} = \frac{10}{182.61} = 0.055$$

$$\Delta\ max. = \frac{P_0}{K} \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}}$$

$$= \frac{49050}{3.33\ x\ 10^9} \frac{1}{\sqrt{(1-0.055^2)^2 + (2\ x0.1x0.055)^2}} = 1.51\ x\ 10^{-5}\ m$$

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(3)

$$f = \pm \frac{N}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_X}{I_X} Y \longrightarrow \text{Normal stress equation}$$

Static loading

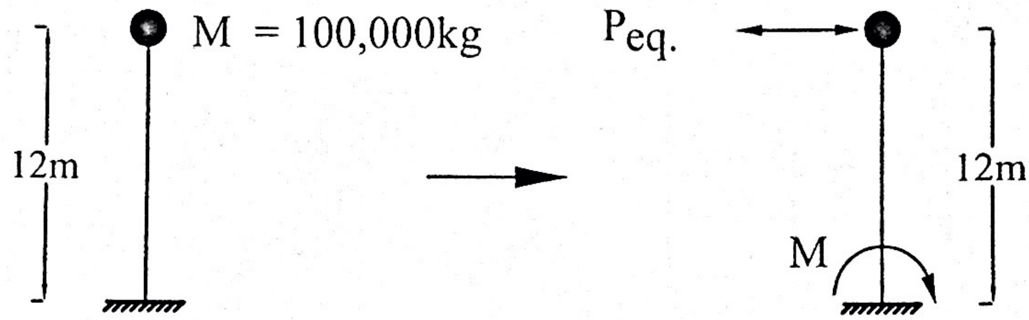
$$f = \frac{-N}{A} = \frac{-981{,}000\ N}{2\ m^2} = -490500\ N/m^2$$

(4) $$f = \frac{N}{A} \pm \frac{M_y}{I_y} X \pm \frac{M_X}{I_X} Y \longrightarrow \text{Normal stress equation}$$

Dynamic loading

Stresses on Column base due to dynamic loading

For (X) direction.

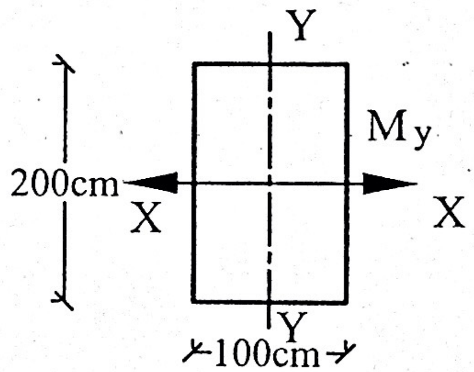


$P_{eq.} = K \times \Delta_{max.}$

$= 5798611 \text{ N/m} \times 0.0179 \text{ m}$

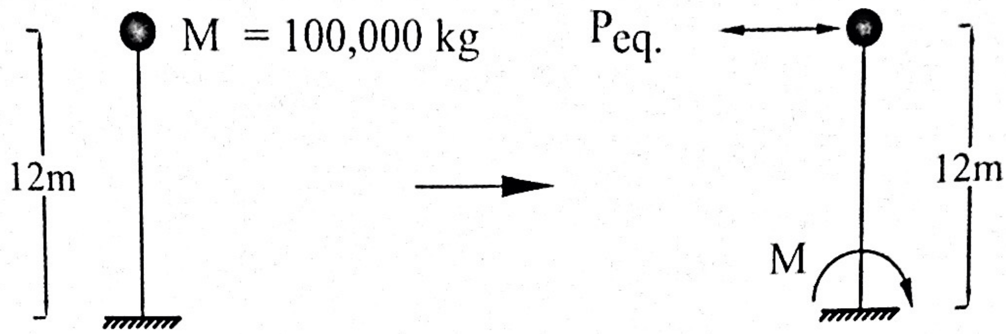
$= 103795 \text{ N}$

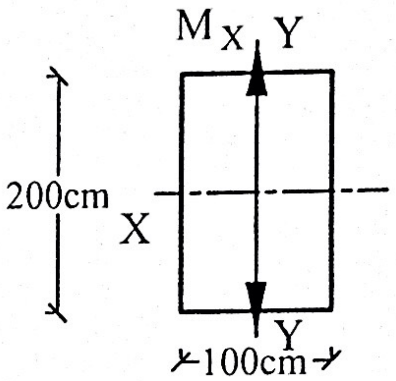
$M = 103795 \times 12 = 1245541 \text{ N.m}$



$$f = \pm \frac{M_y}{I_y} X = \frac{1245541 \text{ N.m}}{0.167 \text{ m}^4} \times 0.5 = \pm 3729166 \text{ N/m}^2$$

For (Y) direction.





$$P_{eq.} = K \times \Delta_{max.}$$

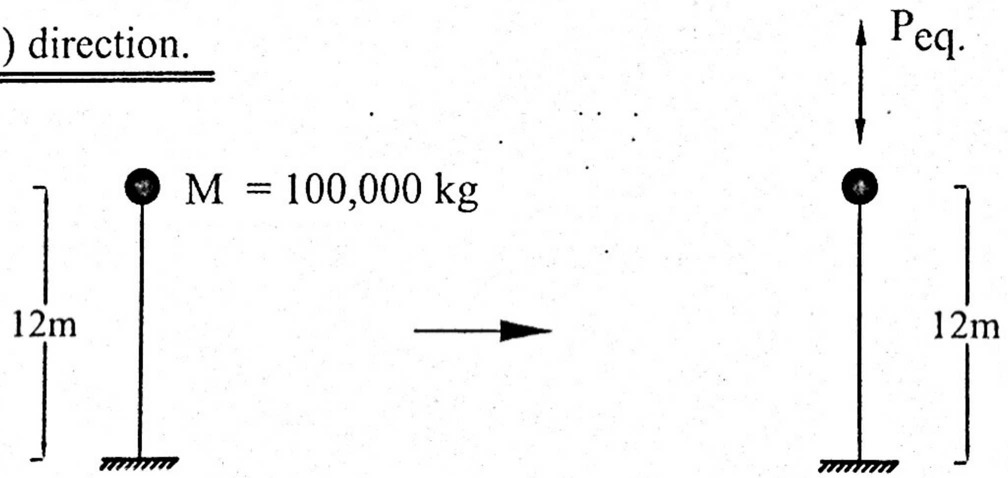
$$= 23159722 \text{ N / m} \times 0.0111 \text{ m}$$

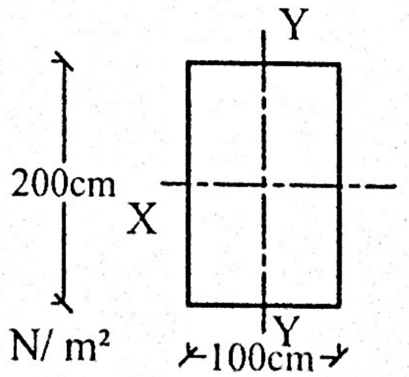
$$= 257072 \text{ N}$$

$$M = 257072 \times 12 = 3084874 \text{ N.m}$$

$$f = \pm \frac{M_x}{I_x} Y = \frac{3084874 \text{ N.m}}{0.667 \text{ m}^4} \times 1 = \pm 4624998 \text{ N/ m}^2$$

For (Z) direction.





$$P_{eq.} = K \times \Delta_{max.}$$

$$= 3.33 \times 10^9 \text{ t/m} \times 1.51 \times 10^{-5} \text{ m}$$

$$= 50333 \text{ N}$$

$$f = \pm \frac{N}{A} = \frac{50333\text{N}}{2 \text{ m}^2} = \pm 25166 \text{ N/ m}^2$$

(5) Combined dynamic stresses :

لا يمكن تجميع الـ Stresses الناتجة من الحركة فى اتجاه X,Y,Z معا لاختلاف الـ Phase angle

أى أننا لا نستطيع أن نقول

$$f = f_x + f_y + f_z$$

و لكن مجازا سوف نقوم بعمل ذلك

Method 1 :

$$f = f_x + f_y + f_z$$

$$= \pm 3729166 \text{ N/m}^2 \pm 4624998 \text{ N/m}^2 \pm 25166 \text{ N/m}^2$$

$$= \pm 8379330 \text{ N/m}^2$$

و هناك حل أخر تقريبى أيضا

Method 2 :

$$f = \pm \sqrt{f_x^2 + f_y^2 + f_z^2} = 5941205 \text{ N/m}^2$$

(6) Total stresses (static + dynamic) :

$$f_{Total} = f_{Static} + f_{dynamic}$$

Using Method 1 :

$$f_{Total} = -490500 \pm 8379330 = -8869830 \ N/m^2 \quad \text{or} \quad 7888830 \ N/m^2$$

Using Method 2 :

$$f_{Total} = -490500 \pm 5941205 = -6431705 \ N/m^2 \quad \text{or} \quad 5450705 \ N/m^2$$

(7) Resonance State

For (X) direction.

$$\beta = \frac{\Omega}{\omega} = 1 \qquad \beta = \frac{10}{\omega} = 1 \qquad \omega = 10 \text{ rad / sec}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{5798611 \text{ N /m}}{M}} = 10$$

M = 57986 kg

Weight = 57986 kg x 9.81 m/sec² = 568843 N

For (Y) direction.

$$\beta = \frac{\Omega}{\omega} = 1 \qquad \beta = \frac{10}{\omega} = 1 \qquad \omega = 10 \text{ rad / sec}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{23159722 \text{ N / m}}{M}} = 10$$

M = 231597 kg

Weight = 231597 kg x 9.81 m/sec² = 2271968 N

For (Z) direction.

$$\beta = \frac{\Omega}{\omega} = 1 \qquad \beta = \frac{10}{\omega} = 1 \qquad \omega = 10 \text{ rad / sec}$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{3.33 \times 10^{9} \text{ t /m}}{M}} = 10$$

$M = 3.33 \times 10^{7}$ kg

Weight = 3.33×10^{7} kg x 9.81 m/sec² = 326673000 N

Minimum weight = 568843 N