FRAMES

#لابد أن نحلها بال *Virtual work method* .

#عند حساب ال *Degree of indeterminacy* لا نهمل القوى فى اتجاه ال X .

#نشتغل بنفس خطوات الكمرات .

1) Once statically indeterminate:

Example:

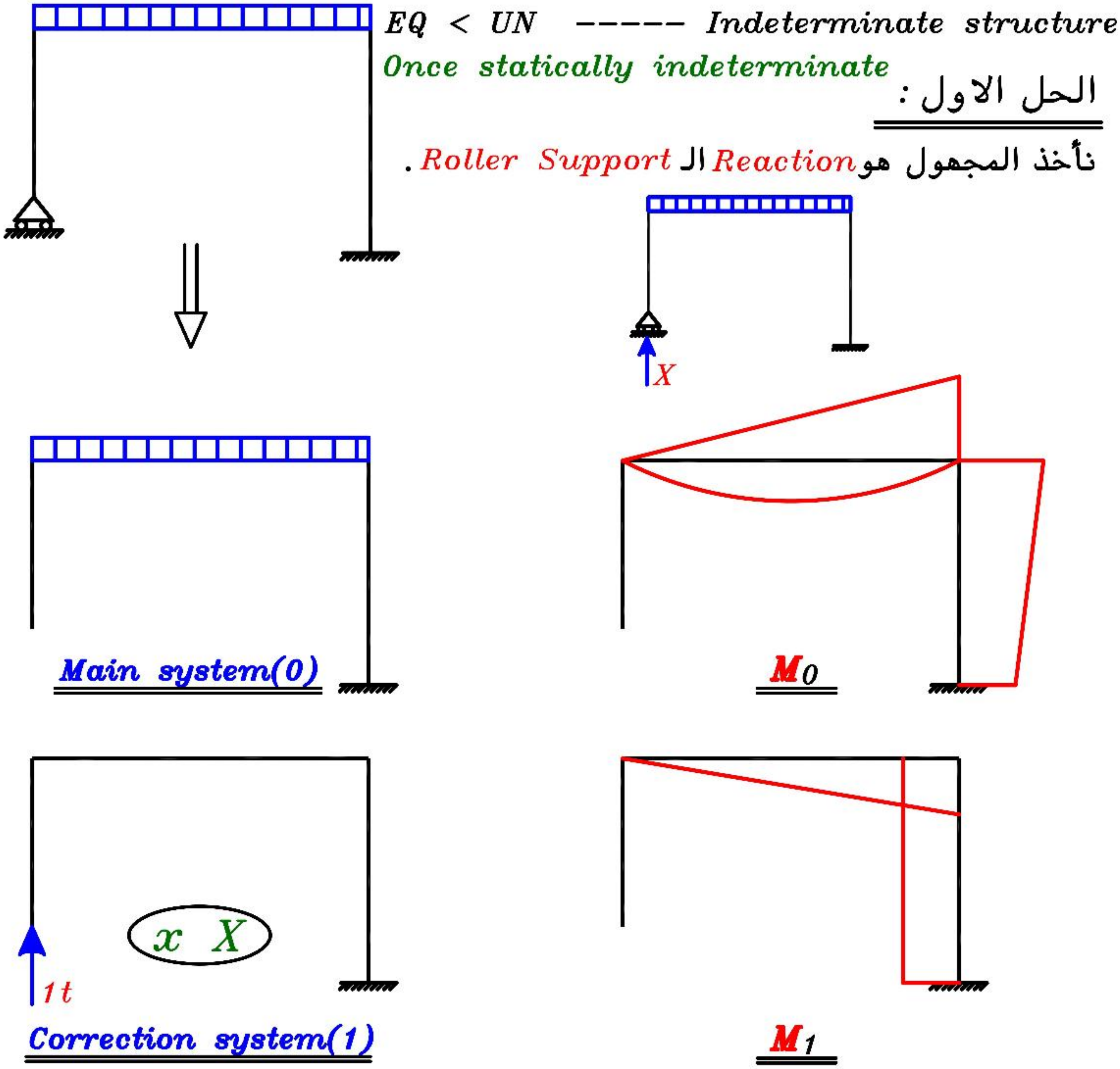
$UN = 4$ & $EQ = 3$

$EQ < UN$ ----- Indeterminate structure

Once statically indeterminate

الحل الاول :

نأخذ المجهول هو *Reaction* ال *Roller Support* .



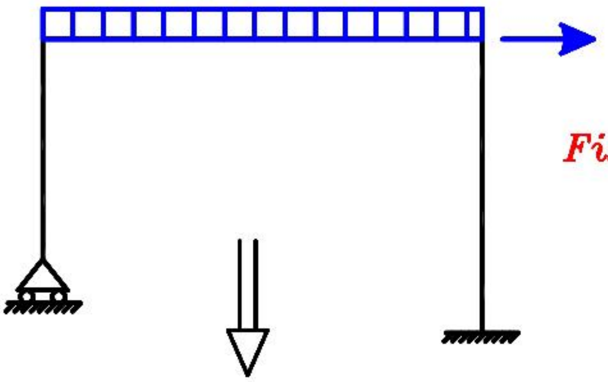
بحل المعادلة نحصل على الـ X $\Longrightarrow$ $\delta_{10} + \delta_{11} x X = 0$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \qquad \delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$\boxed{M_{final} = M_0 + (X) M_1} \qquad \boxed{R_{final} = R_0 + (X) R_1}$$

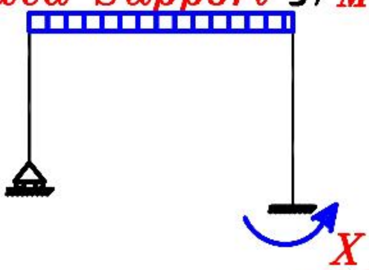
خد باك

الـ δ المحسوبة فى هذا الحل هى *Vertical deflection* لاننا قمنا بازالة *Vertical Reaction* ولكن فى النهاية لا يفرق نوع الـ δ لاننا دائما نضرب *moment* فى *moment*.

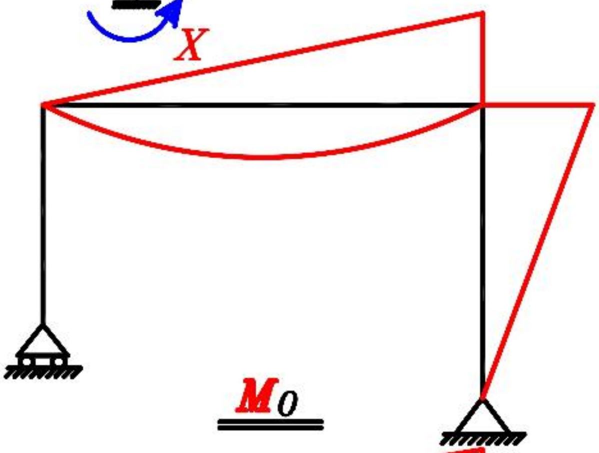


الحل الثانى:

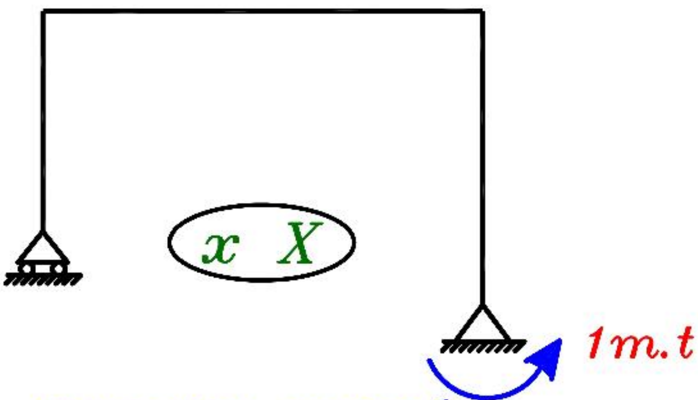
نأخذ المجهول هو الـ *Moment* الـ *Fixed Support*



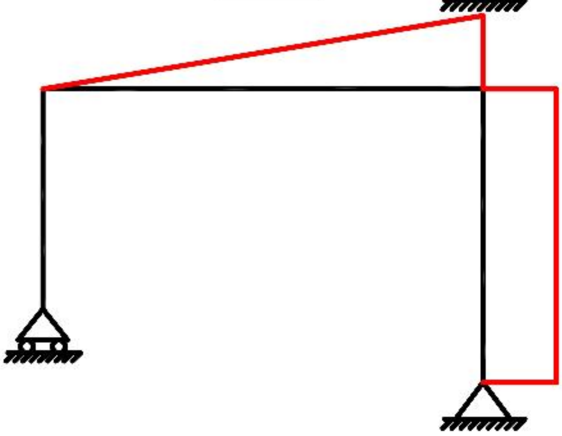
Main system(0)



M_0



Correction system(1)



M_1

$\delta_{10} + \delta_{11} x X = 0 \Longrightarrow$ بحل المعادلة نحصل على ال X

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \qquad \delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$\boxed{M_{final} = M_0 + (X) M_1} \qquad \boxed{R_{final} = R_0 + (X) R_1}$$

خد باك

الـ δ المحسوبة فى هذا الحل هى *Vertical deflection* لاننا قمنا بازالة *Rotation* ولكن فى النهاية لا يفرق نوع الـδ لاننا دائما نضرب *moment* فى *moment*.

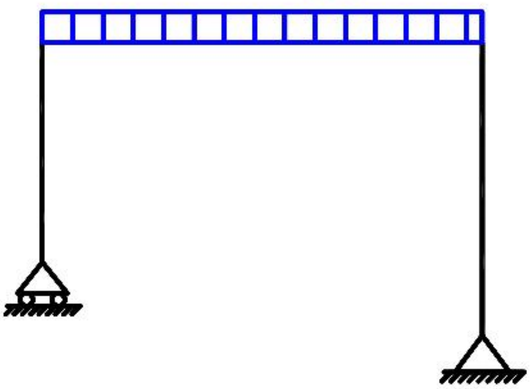
2) Twice statically indeterminate:

UN = 5 & EQ = 3

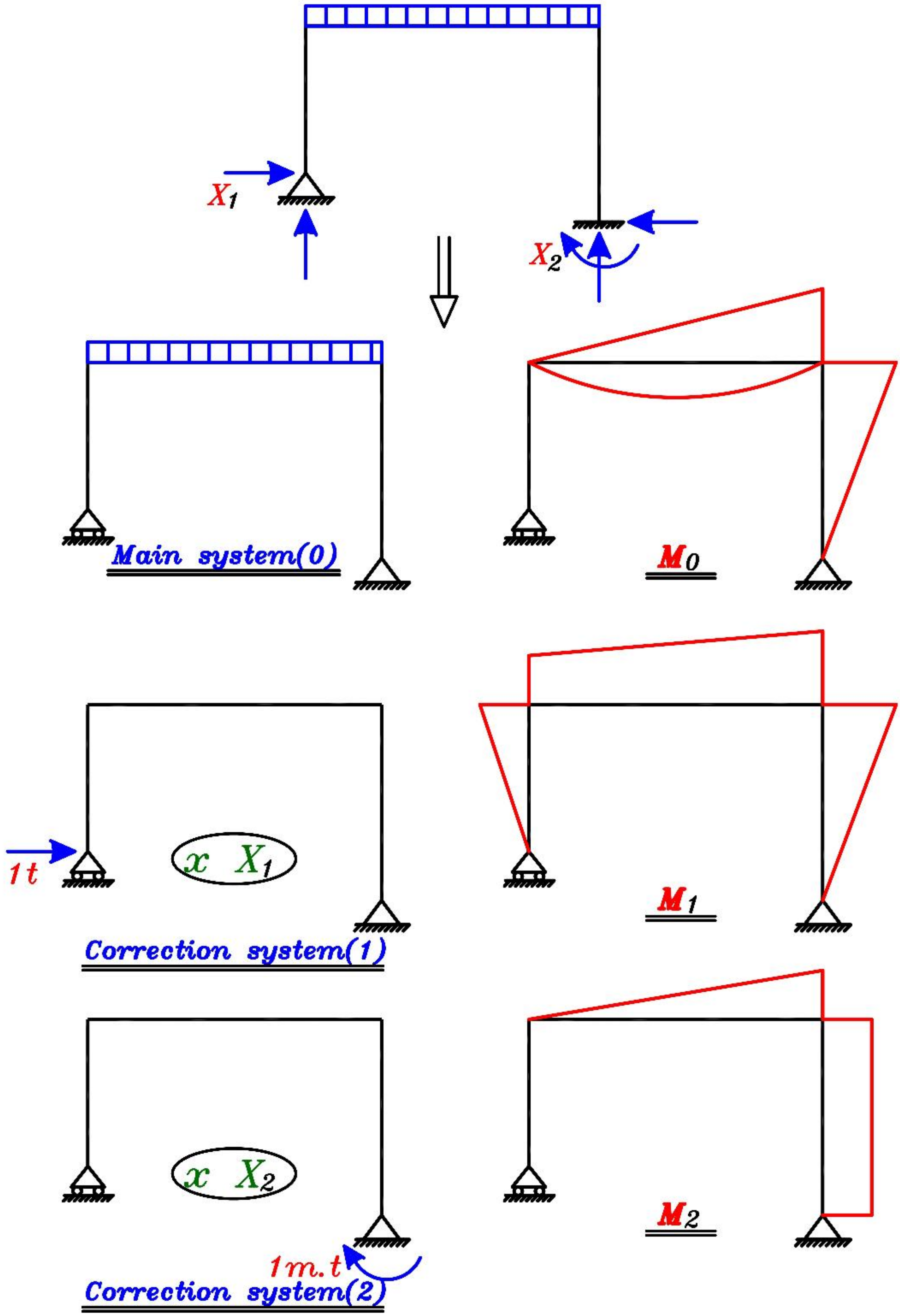
EQ < UN ------ Indeterminate structure

Twice statically indeterminate

نأخذ المجاهيل هى *Moment* الـ *Fixed Support* و الـ *Horizontal Reaction* للـ *Hinged Support*.



من الممكن أخذ أى مجهولين أخرين ولكن بشرط أن يظل الـ *System* $\Longleftarrow$ *Stable* و يصبح *Determinate*.


X_1
X_2
Main system(0)
M_0
1t
x X_1
M_1
Correction system(1)
x X_2
1m.t
Correction system(2)
M_2

$$\delta_{10} + \delta_{11} x X_1 + \delta_{12} x X_2 = 0$$
$$\delta_{20} + \delta_{21} x X_1 + \delta_{22} x X_2 = 0$$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \quad \delta_{11} = \int \frac{M_1 M_1}{EI} dL \quad \delta_{12} = \int \frac{M_1 M_2}{EI} dL$$
$$\delta_{20} = \int \frac{M_2 M_0}{EI} dL \quad \delta_{21} = \int \frac{M_2 M_1}{EI} dL \quad \delta_{22} = \int \frac{M_2 M_2}{EI} dL$$

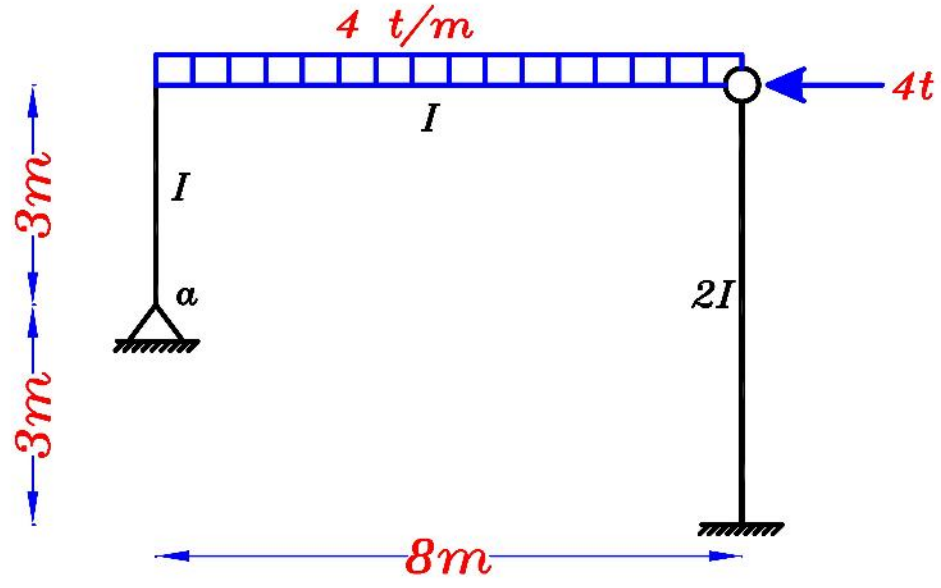
$$\boxed{\delta_{21} = \delta_{12}}$$

$$\boxed{M_{final} = M_0 + (X_1) M_1 + (X_2) M_2}$$

$$\boxed{R_{final} = R_0 + (X_1) R_1 + (X_2) R_2}$$

Example:

For the shown frame draw the B.M.D .



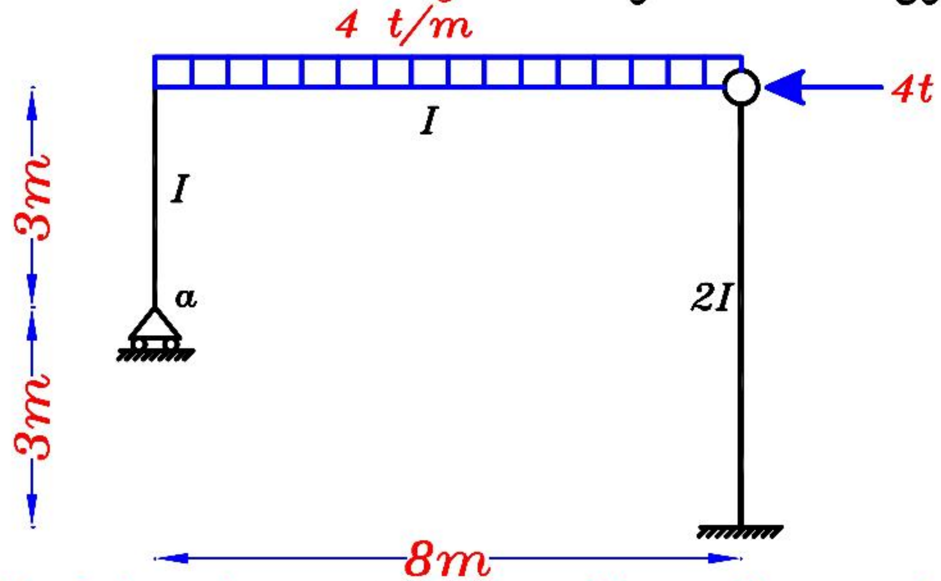
١- نحدد المعادلات و المجاهيل و نحدد ***degree of indeterminacy*** .

$UN = 5 \quad \& \quad EQ = 4$

$EQ < UN$ ----- *Indet. structure*

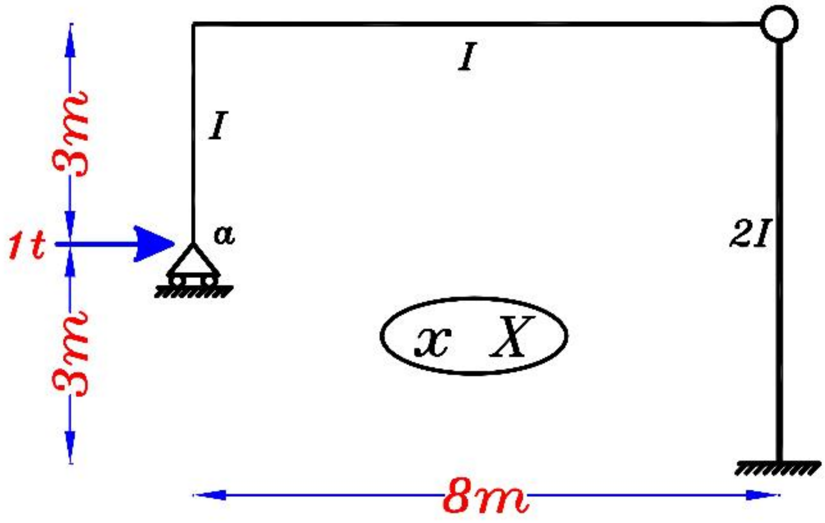
$UN - EQ = 5 - 4 = 1$

٢- نزيل المجاهيل الزائدة عن المعادلات بحيث يصبح المنشأ ***determinate*** و ***Stable*** و يكون هذا المنشأ هو الـ ***Main system*** .



٣- نكون الـ ***Correction systems*** وهم بعدد الـ ***degree of indeterminacy***

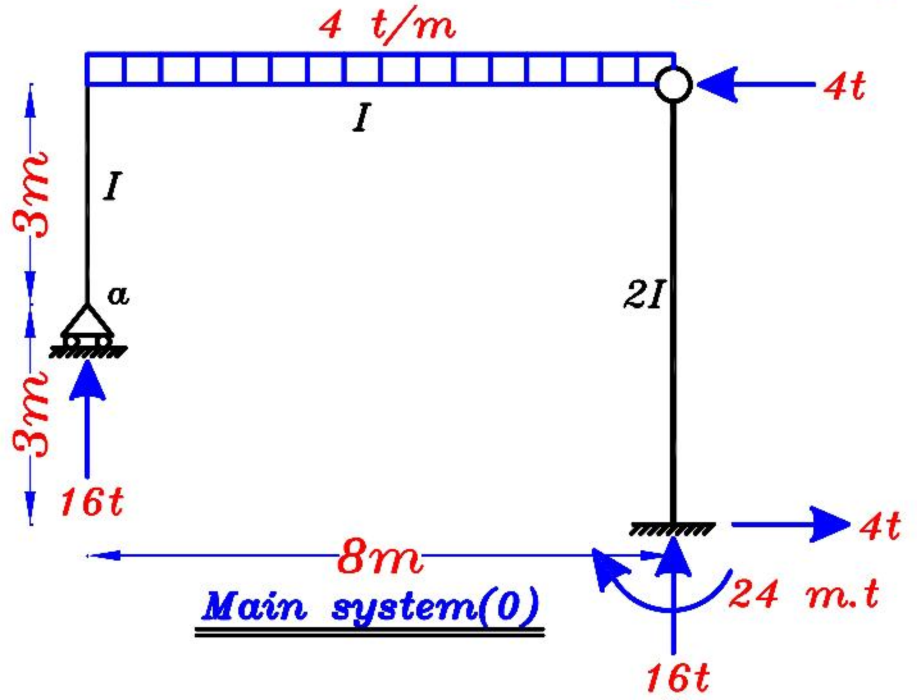
و أى ***Correction system*** هو عبارة عن الـ ***Main system*** مُزال من عليه الـ ***Loads*** و موضوع عليه أحد المجاهيل المزالة .

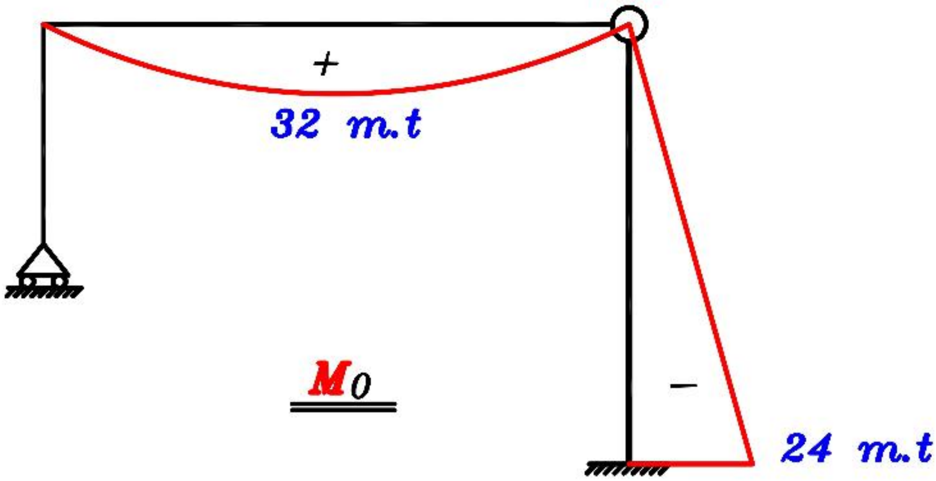


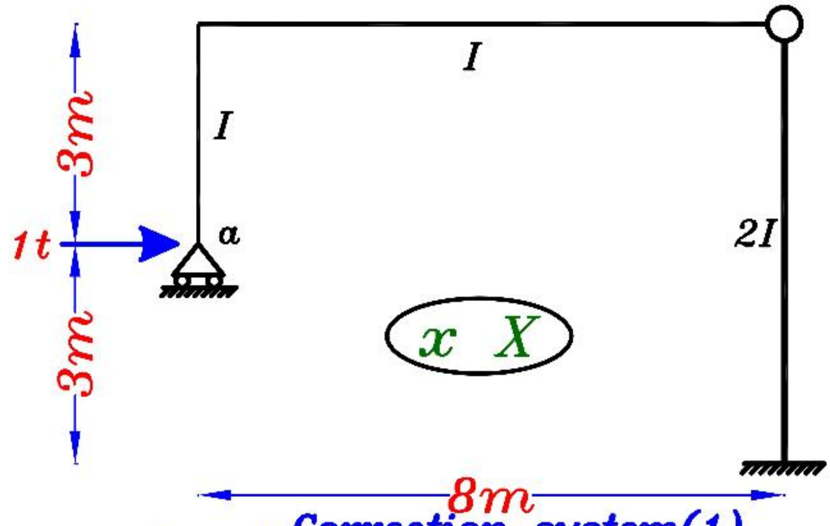
٤- نكتب معادلة الـ Consistent Deformation لكل مجهول مُزال

$$\delta_1 = \delta_{10} + \delta_{11} x X = 0$$

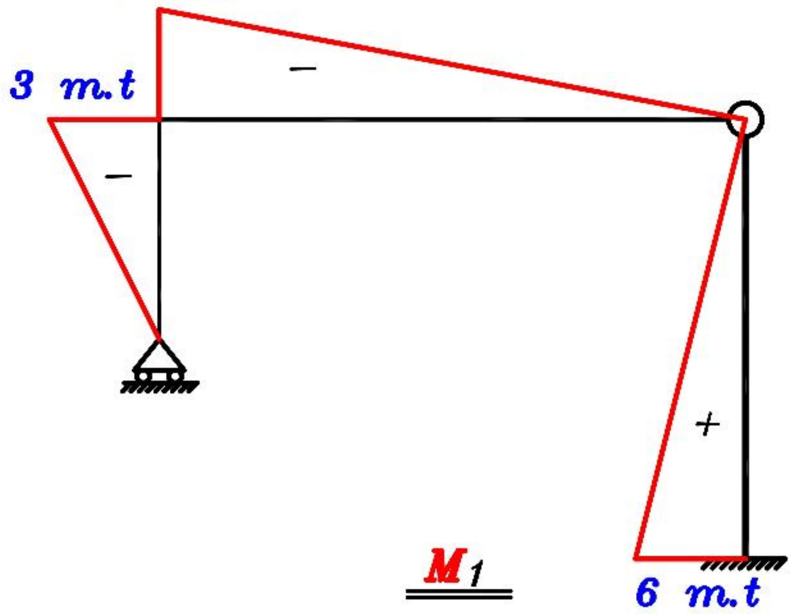
٥- نرسم الـ B.M.D للـ Main system و نسميه M_0 و للـ Correction system (1) و نسميه M_1 و للـ Correction system (2) و نسميه M_2 و هكذا .







Correction system(1)



M_1

٦- نحسب الـDeformations التى حددناها من الخطوة السابقة عن طريق ضرب الـ B.M.D لرقمى الـ Deformation .

$$\delta_{10} = \int \frac{M_1\, M_0}{EI}\, dL \qquad \delta_{11} = \int \frac{M_1\, M_1}{EI}\, dL$$

$$\delta_{10} = \frac{-1}{2EI}\left[(1/2x24x6)\,(2/3x6)\right] - \frac{1}{EI}\left[(2/3x8x32)\,(1.5)\right]$$

$$= \frac{-400}{EI}$$

$$\delta_{11} = \frac{1}{2EI}\left[\,(1/2x6x6)\;(2/3x6)\right] + \frac{1}{EI}\left[(1/2x3x8)\;\;(2/3x3)\right.$$
$$\left.+\;(1/2x3x3)\,(2/3x3)\,\right]$$

$$= \frac{69}{EI}$$

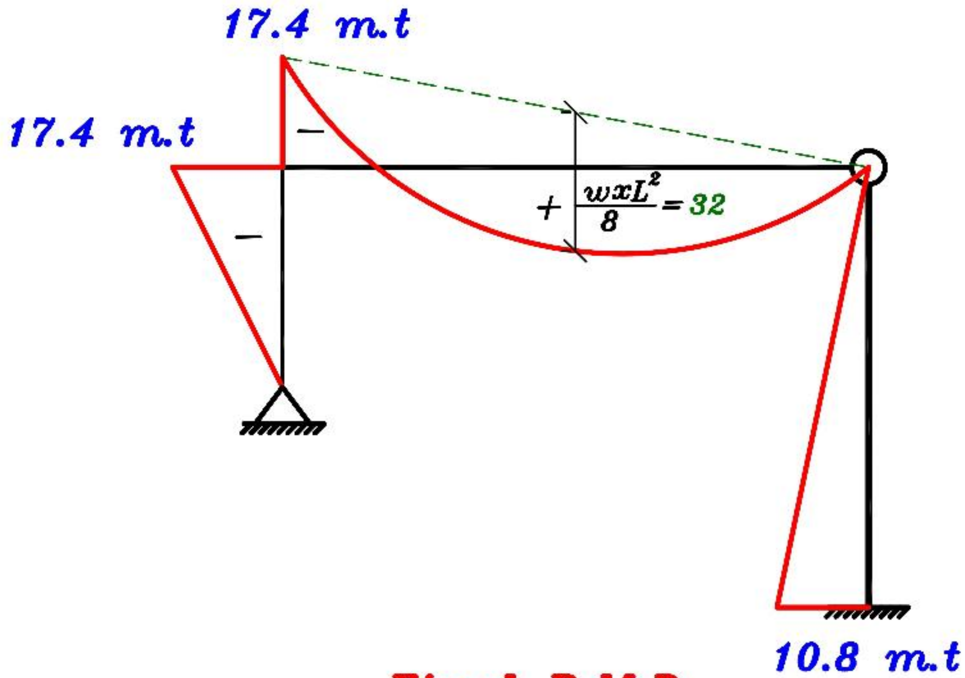
٧- نحل المعادلات معا و نحصل على المجاهيل .

$$\delta_{10} + \delta_{11}\, x\, X = 0$$

$$\frac{-400}{EI} + \frac{69}{EI}\, x\, X = 0 \Longrightarrow \boxed{X = 5.8t}$$

٨- نحساب الـ*Reaction* أو الـ*B.M.D* للكمرة الاصلية .

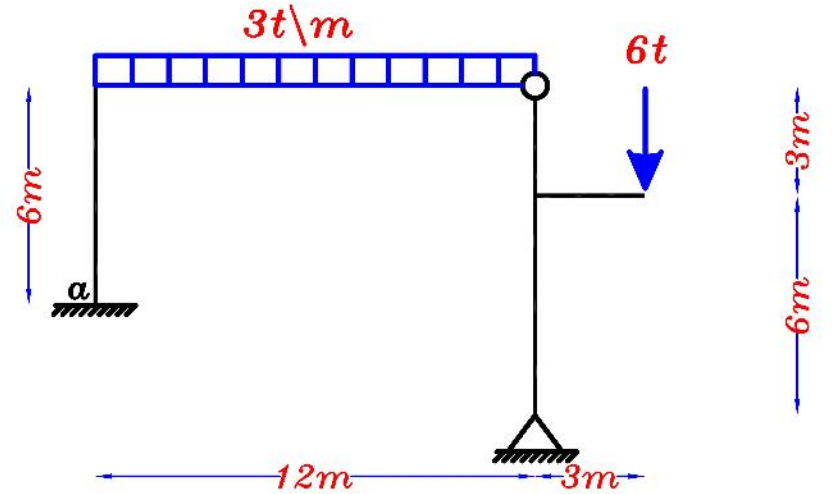
$$\boxed{M_{final} = M_0 + (5.8)M_1}$$



Final B.M.D

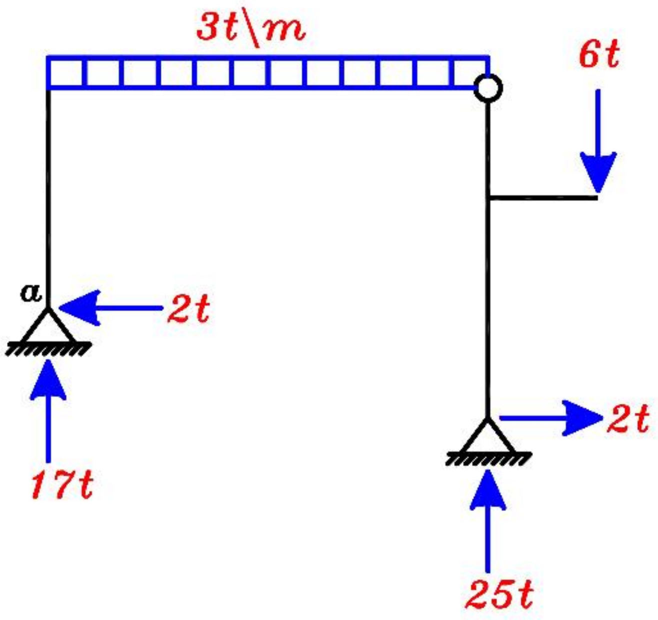
Example:

For the shown frame draw the B.M.D .

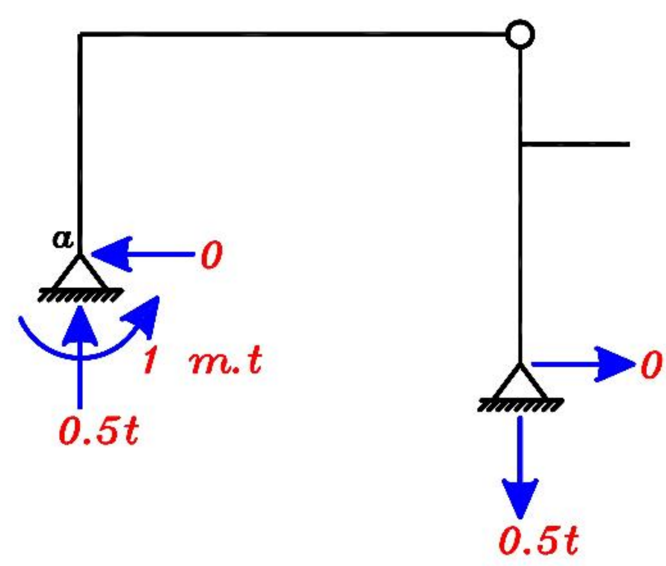


UN = 5 & EQ = 4 EQ < UN ----- Indet. structure

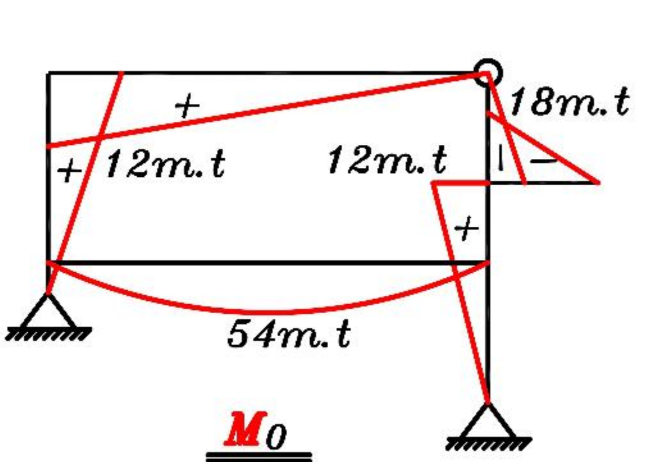
UN − EQ = 5 − 4 = 1



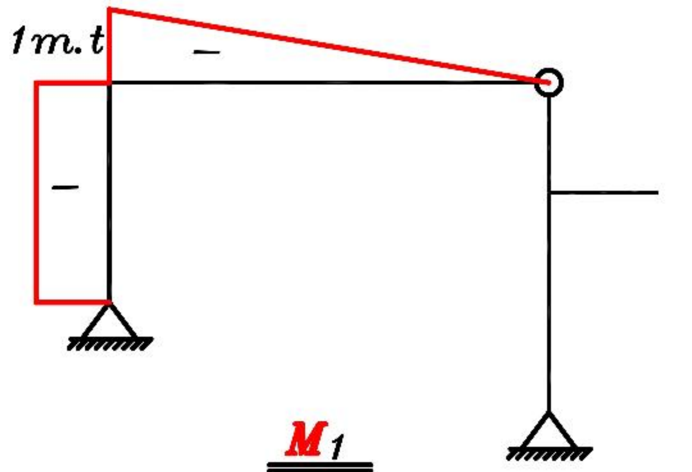
Main system(0)



Correction system(1)



M_0



M_1

$$\delta_{10} + \delta_{11} x X = 0$$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \qquad \delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$\delta_{10} = \frac{-1}{EI}[(1/2x12x6)(1) + (1/2x12x12)(2/3x1)]$$

$$+ (2/3x12x54)(1/2x1)] = \frac{-300}{EI}$$

$$\delta_{11} = \frac{1}{EI}[(1/6x1)(1) + (1/2x12x1)(2/3x1)]$$

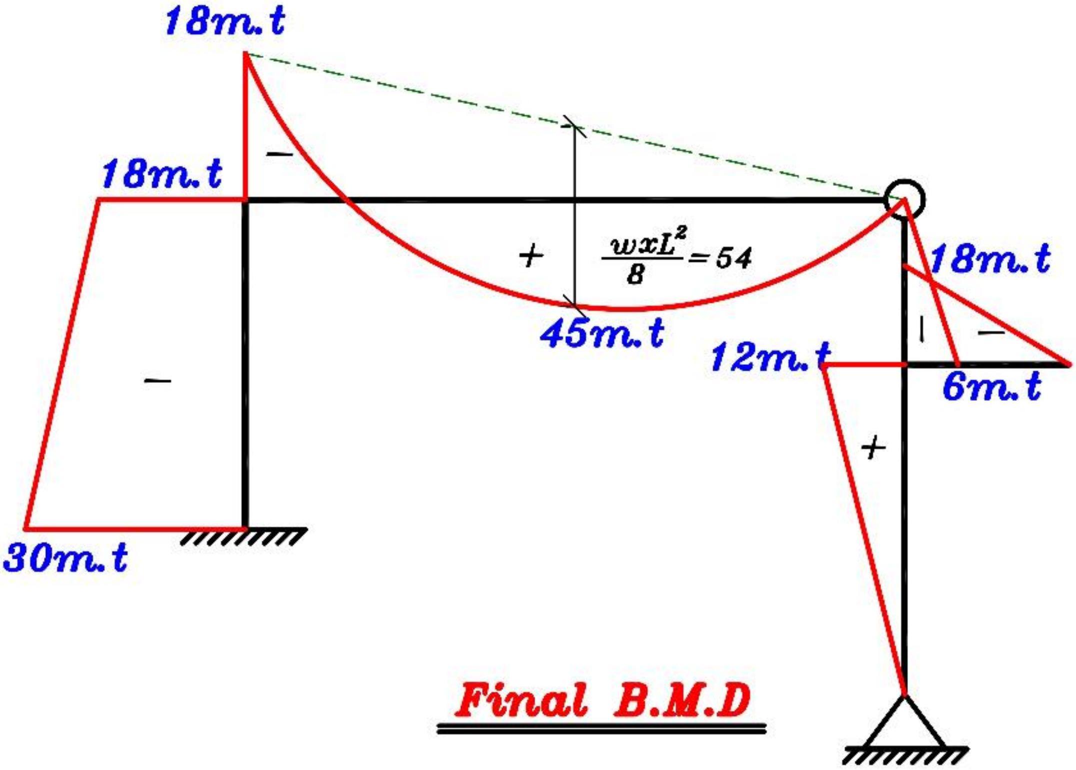
$$= \frac{10}{EI}$$

$$\delta_{10} + \delta_{11} x X = 0$$

$$\frac{-300}{EI} + \frac{10}{EI} x X = 0 \implies \boxed{X = 30m.t}$$

$$\boxed{M_{final} = M_0 + (30) M_1}$$

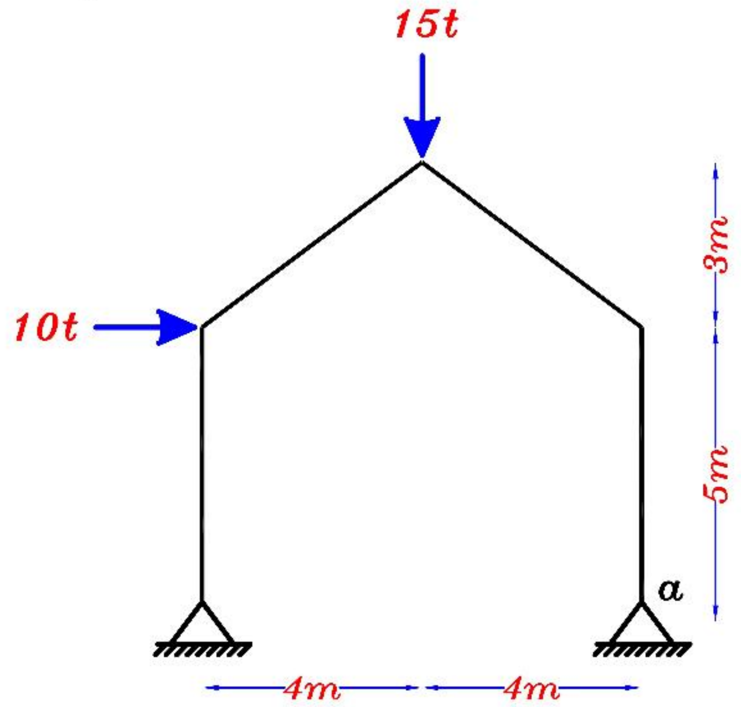
$$\boxed{R_{final} = R_0 + (30) R_1}$$



Final B.M.D

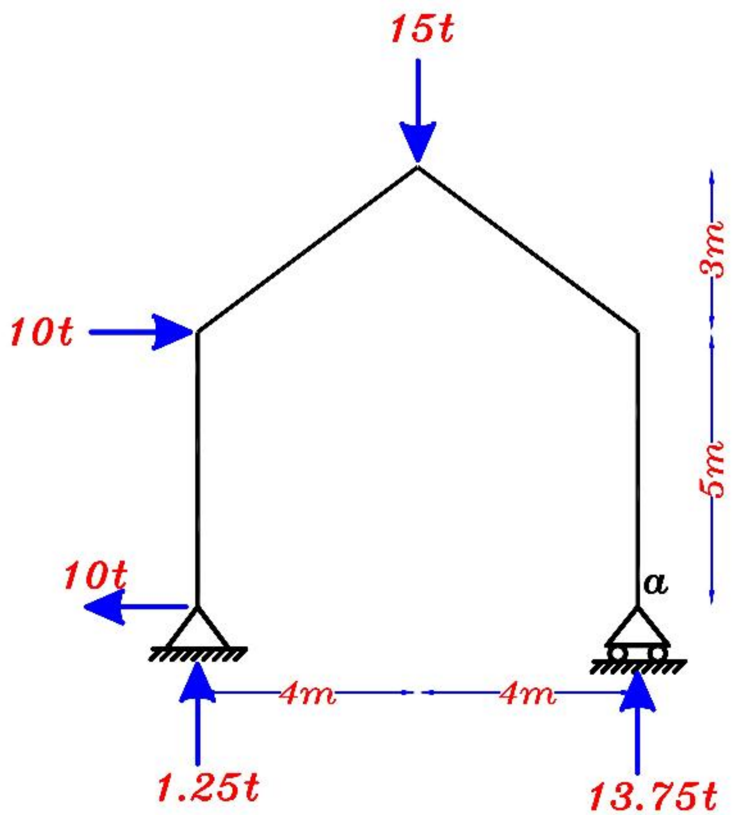
Example:

For the shown frame draw the B.M.D .

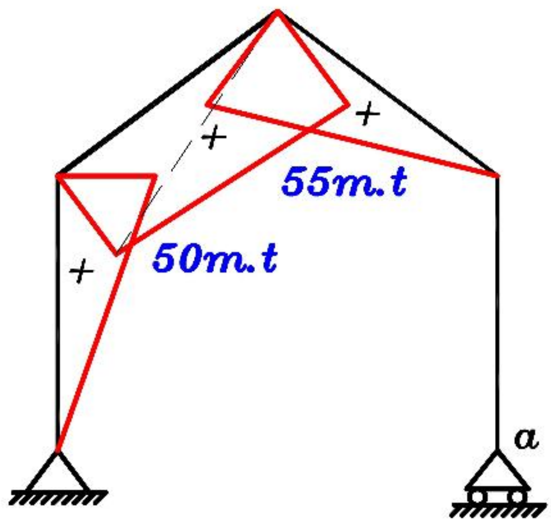


UN = 4 & EQ = 3 EQ < UN ----- Indet. structure

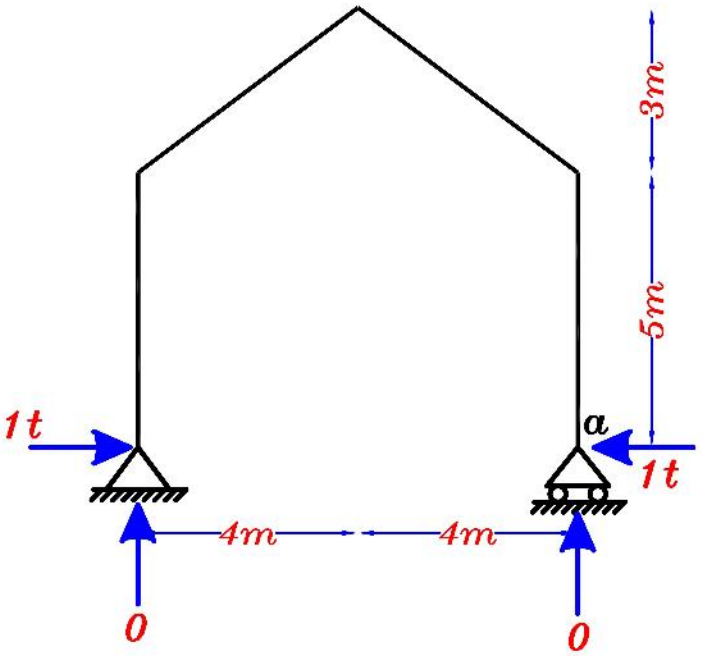
UN − EQ = 4 − 3 = 1



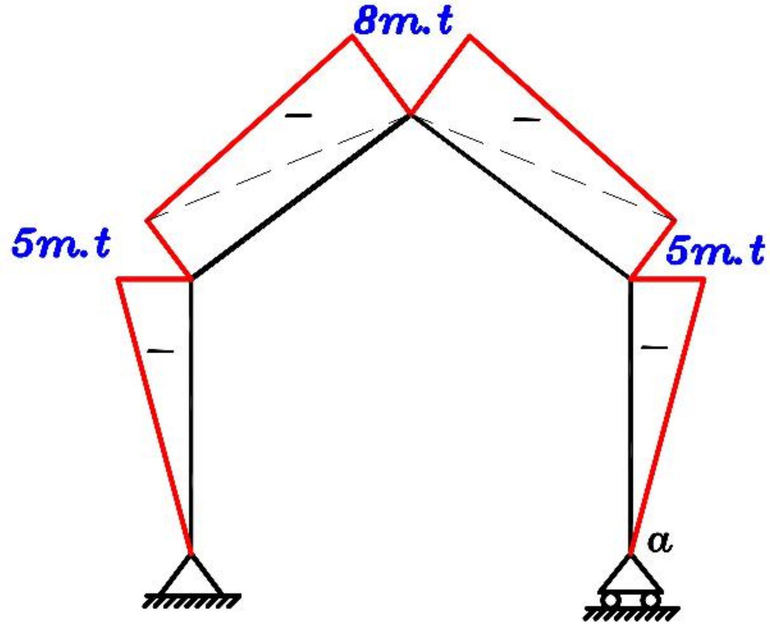
Main system(0)



M_0



Correction system(1)



M_1

$$\delta_{10} + \delta_{11} x X = 0$$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \qquad \delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$\delta_{10} = \frac{-1}{EI} [(1/2x5x5)(2/3x50) + (1/2x5x5)(2/3x50+1/3x55)$$

$$+ (1/2x5x8)(2/3x55+1/3x50) + (1/2x5x55)(2/3x8+1/3x5)]$$

$$= \frac{-3091.6}{EI}$$

$$\delta_{11} = \frac{2}{EI} [(1/2x5x5)(2/3x5+1/3x8) + (1/2x5x8)(2/3x8+1/3x5)$$

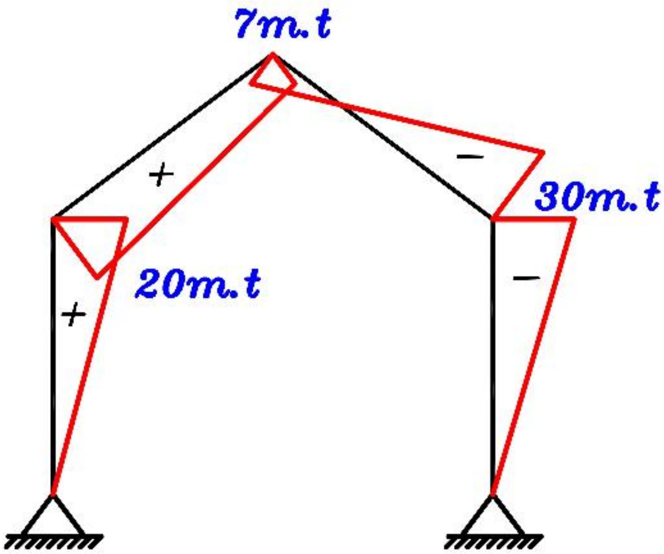
$$+ (1/2x5x5)(2/3x5) = \frac{513.33}{EI}$$

$$\delta_{10} + \delta_{11} x X = 0$$

$$\frac{-3091.6}{EI} + \frac{513.33}{EI} x X = 0 \implies \boxed{X = 6.02t}$$

$$\boxed{M_{final} = M_0 + (6.02)M_1}$$

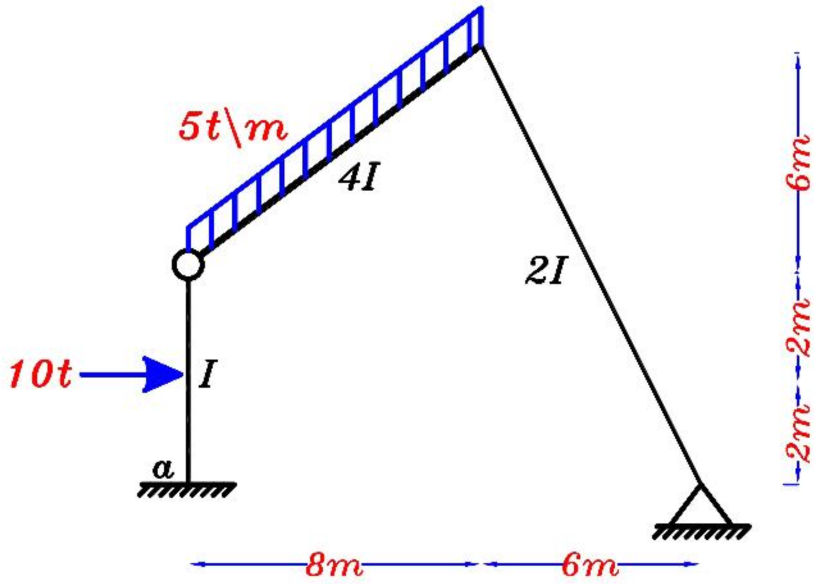
$$\boxed{R_{final} = R_0 + (6.02)R_1}$$



Final B.M.D

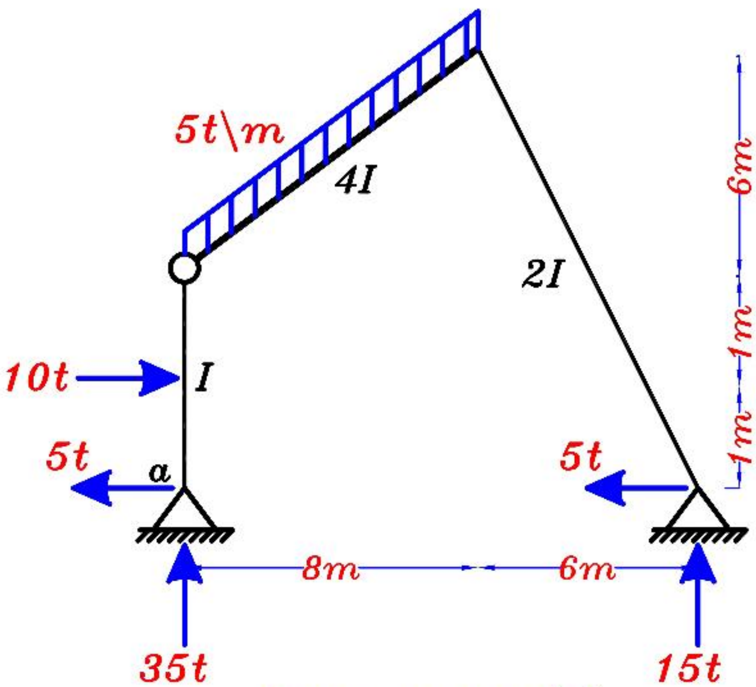
Example:

For the shown frame draw the B.M.D .

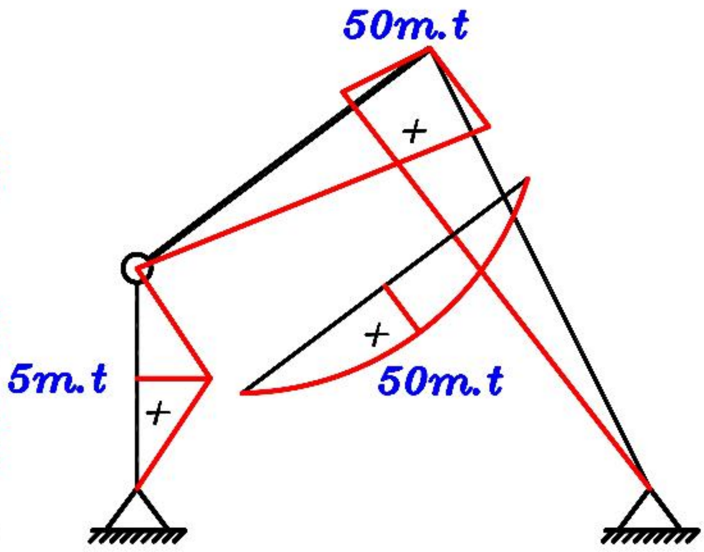


UN = 5 & EQ = 4 EQ < UN ----- Indet. structure

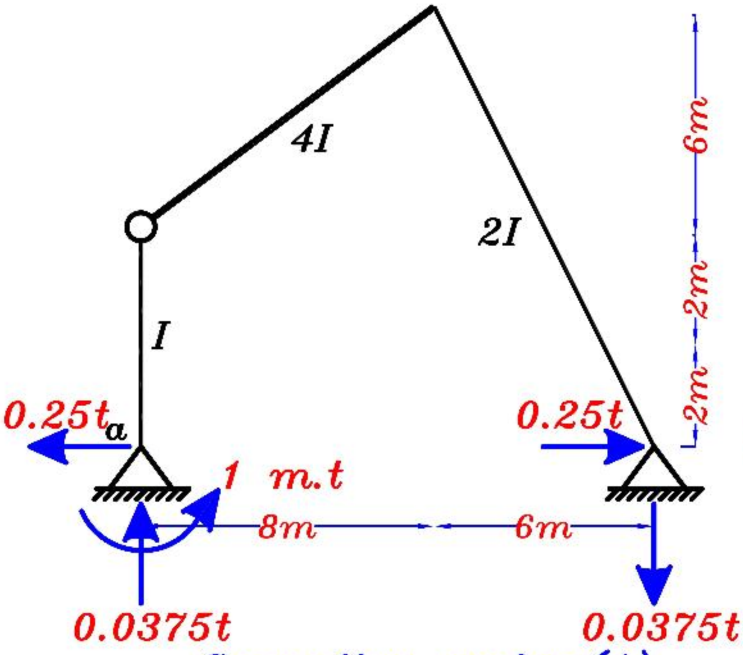
UN − EQ = 5 − 4 = 1



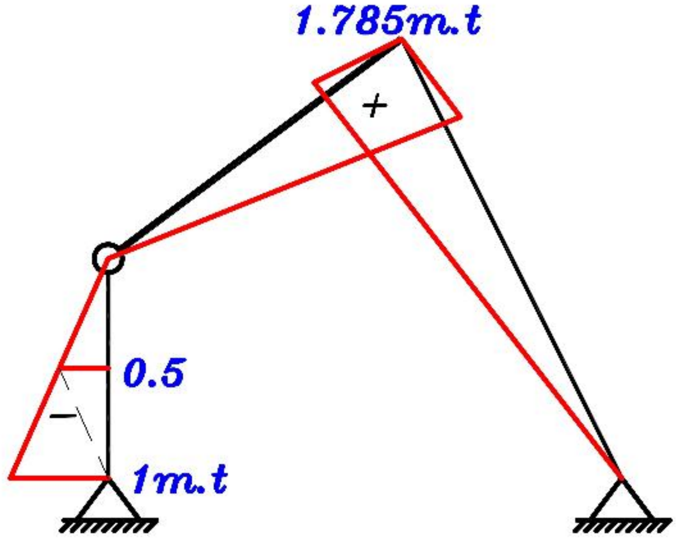
Main system(0)



M_0



Correction system(1)



M_1

$$\delta_{10} + \delta_{11} x X = 0$$

$$\delta_{10} = \int \frac{M_1 \, M_0}{EI} dL \qquad \delta_{11} = \int \frac{M_1 \, M_1}{EI} dL$$

$$\delta_{10} = \frac{-1}{EI}[(1/2x1x5) \ (2/3x0.5+1/3x1) + (1/2x1x5) \ (2/3x0.5)]$$
$$+ \frac{1}{4EI} (1/2x10x50)(2/3x1.785) + (2/3x10x50)(1/2x1.785)]$$
$$+ \frac{1}{2EI} (1/2x10x50)(2/3x1.785) = \frac{272.44}{EI}$$

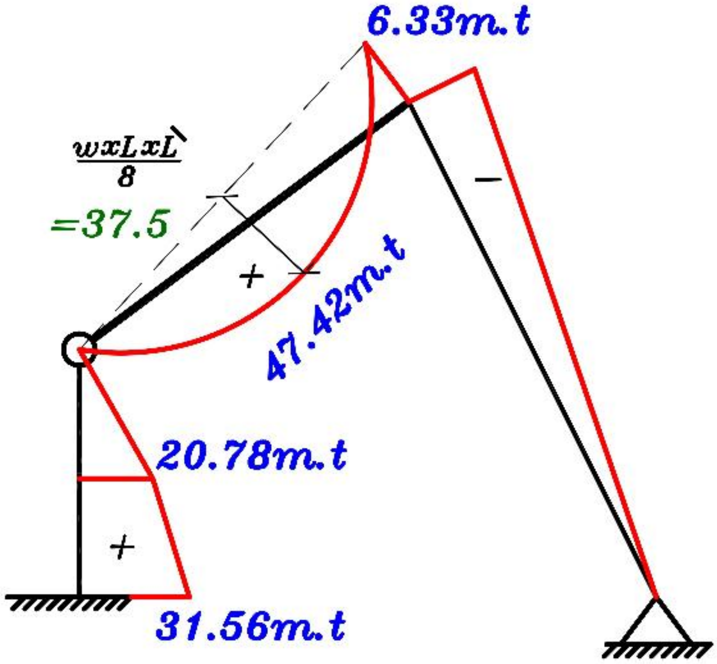
$$\delta_{11} = \frac{1}{EI}[(1/2x2x1) \ (2/3x1)]$$
$$+ \frac{1}{4EI} (1/2x10x1.785)(2/3x1.785)]$$
$$+ \frac{1}{2EI} (1/2x10x1.785)(2/3x1.785) = \frac{8.632}{EI}$$

$$\delta_{10} + \delta_{11} x X = 0$$

$$\frac{272.44}{EI} + \frac{8.632}{EI} \ x X = 0 \Longrightarrow X = -31.56 m.t$$

$$\boxed{M_{final} = M_0 + (-31.56)M_1}$$

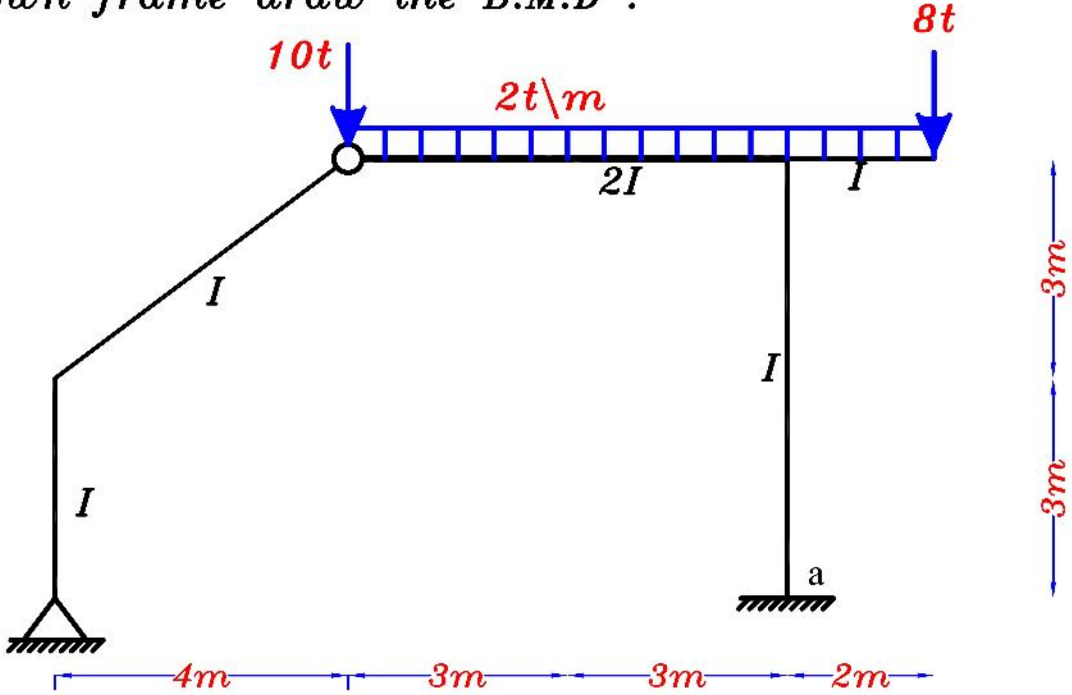
$$\boxed{R_{final} = R_0 + (-31.56)R_1}$$



Final B.M.D

Example:

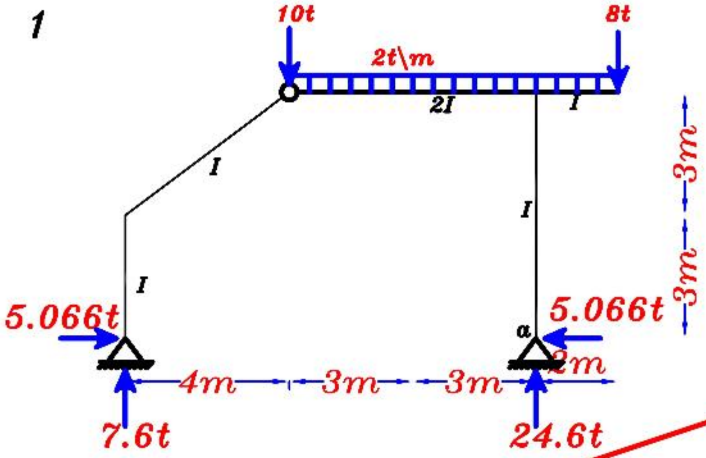
For the shown frame draw the B.M.D .

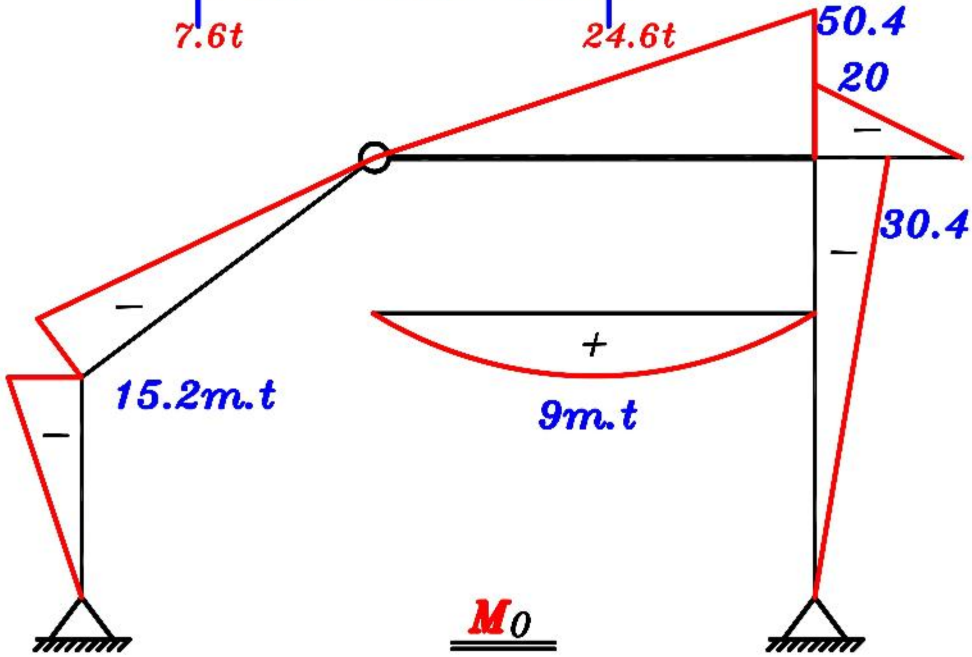


UN = 5 & EQ = 4 EQ < UN ----- Indet. structure

UN − EQ = 5 − 4 = 1

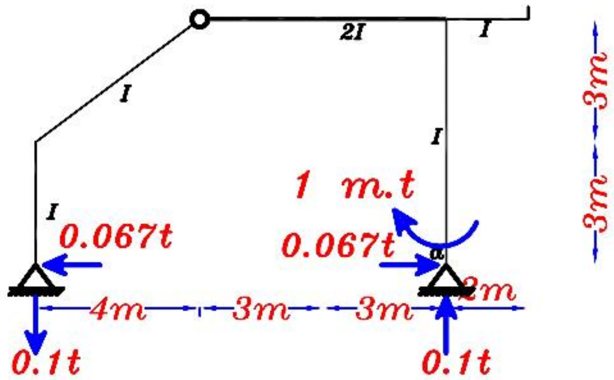
Main system(0)

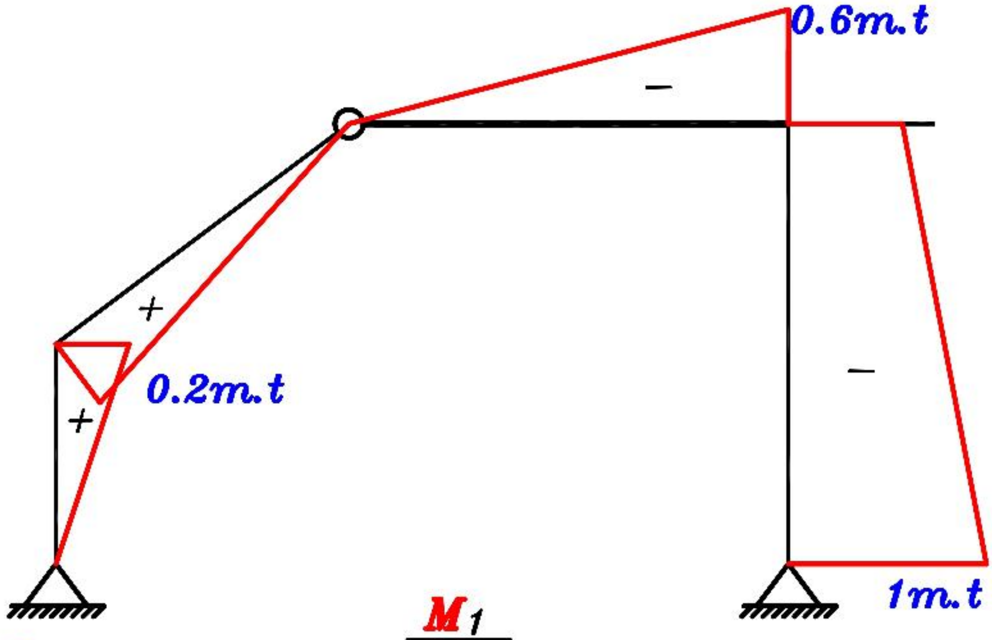




M_0

Correction system(1)





M_1

$$\delta_{10} + \delta_{11} x X = 0$$

$$\delta_{10} = \int \frac{M_1\, M_0}{EI} dL \qquad \delta_{11} = \int \frac{M_1\, M_1}{EI} dL$$

$$\delta_{10} = \frac{1}{EI}[(1/2x3x15.2)(2/3x{-0.2}) + (1/2x5x15.2)(2/3x{-0.2})$$

$$+ (1/2x6x30.4)(2/3x0.6+1/3x1)]$$

$$+ \frac{1}{2EI}[(1/2x6x50.4)(2/3x0.6) + (2/3x6x9)(1/2x{-0.6})]$$

$$= \frac{83.613}{EI}$$

$$\delta_{11} = \frac{1}{EI}[(1/2x3x0.2)(2/3x0.2) + (1/2x5x0.2)(2/3x0.2)$$

$$+ (1/2x6x1)(2/3x1+1/3x0.6) + (1/2x6x0.6)(2/3x0.6+1/3x1)]$$

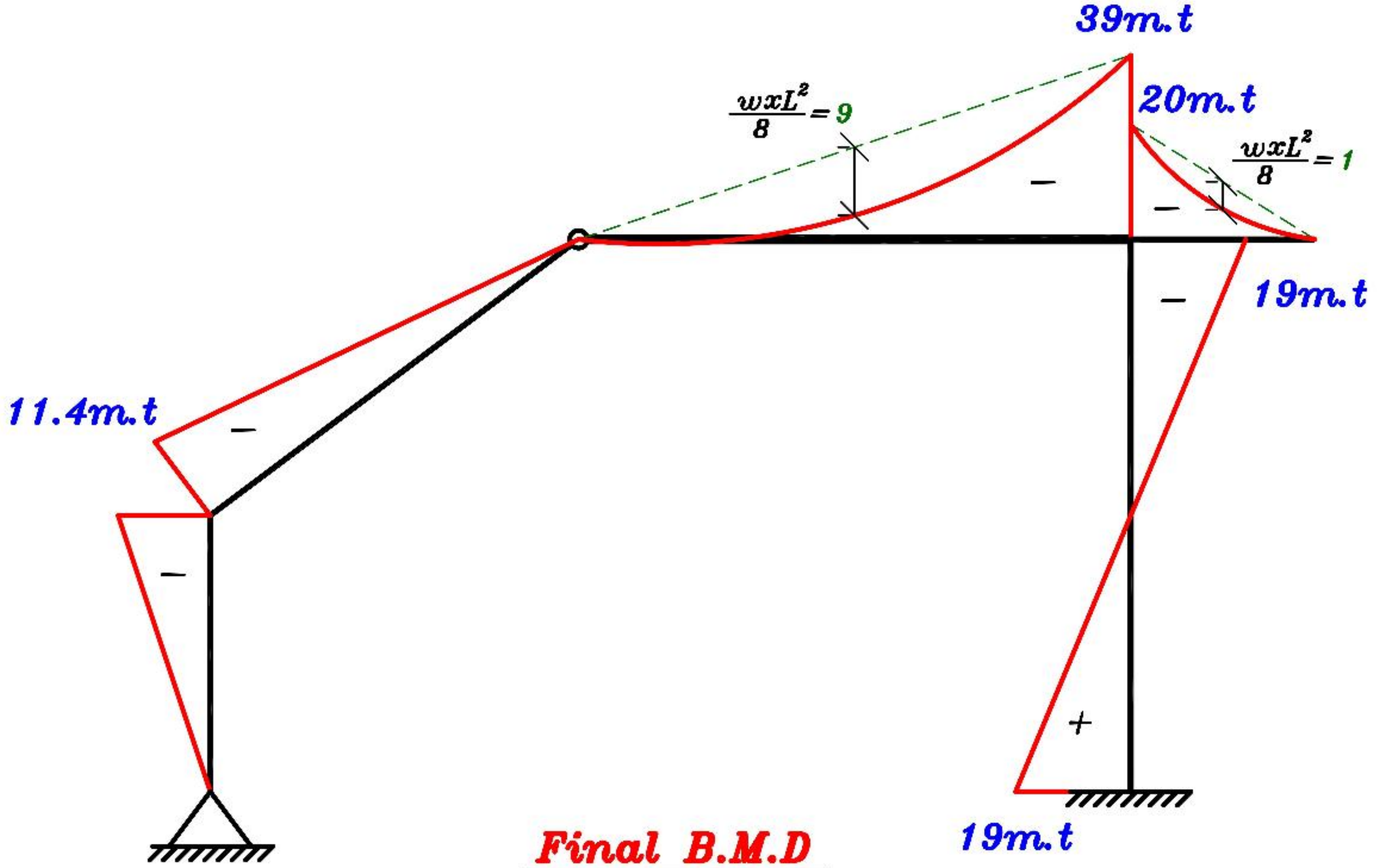
$$+ \frac{1}{2EI}[(1/2x6x0.6)(2/3x0.6)] = \frac{4.3867}{EI}$$

$$\delta_{10} + \delta_{11} x X = 0$$

$$\frac{83.613}{EI} + \frac{4.3867}{EI} x X = 0 \Longrightarrow \boxed{X = -19 m.t}$$

$$\boxed{M_{final} = M_0 + (-19) M_1}$$

$$\boxed{R_{final} = R_0 + (-19) R_1}$$



Final B.M.D

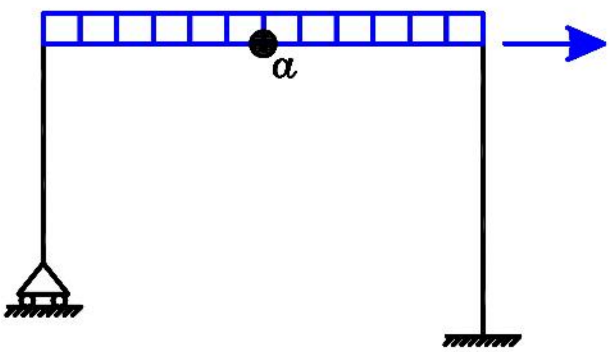
طريقة أخرى للحل

من اللمكن عند حل المسائل بال *Consistent deformations method* بدلا من ازالة مجهول أو أكثر ليصبح المنشأ *Determinate* أن نقوم باضافة معادلة أو أكثر ليصبح المنشأ *Determinate*.

و لزيادة المعادلات نختار أى نقط و نضع عندها *Intrmediate hinges* بحيث نزود عدد المعادلات المطلوبة.

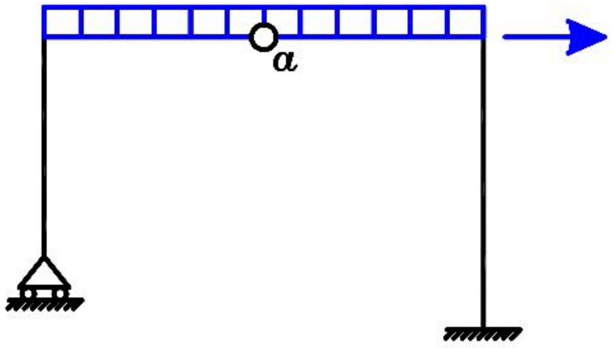
أى نقطة يكون ال *Slope angle* عندها من اليمين تساوى الـ *Slope angle* من اليسار و عند وضع *I.H.* عند هذه النقطة تصبح الـ *Slope angle* من اليمين لا تساوى الـ *Slope angle* من اليسار و بالتالى لحل هذه المسألة تكون فكرتها هى حساب ال *Change in Slope angle* فى الـ *Main system* و فى الـ *Correction* و يكون مجموعهم يساوى المنشأ الاصلى أى يساوى صفر.

Example:

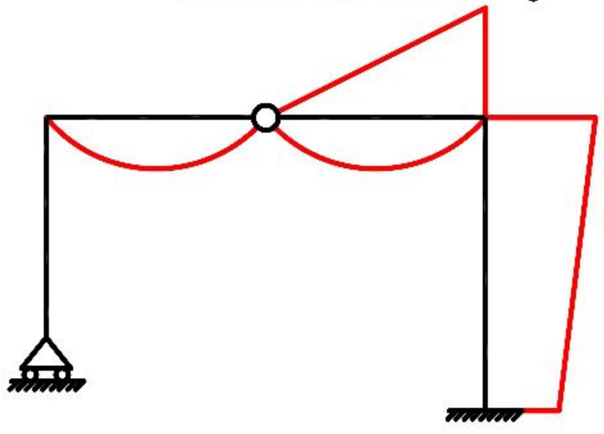


Indeterminate Structure

١- نختار أى نقطة نضع عندها الـ *I.H.* وليس شرط أن تكون فى المنتصف و يكون هذا هو الـ *Main system*.

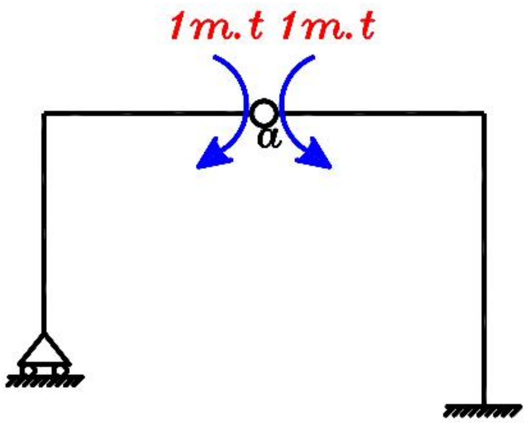


Main system
Determinate&Stable

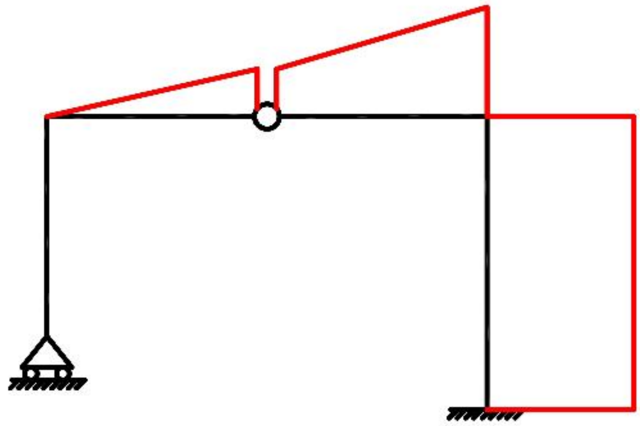


M_0

٢- نكون الـ *Correction system* عن طريق ازالة الـ *Loads* ووضع *2-Moments* عكس بعض فى الاتجاه عند الـ *I.H.* لاننا نريد حساب *Change in Slope angle*



Correction system



$\boldsymbol{M_1}$

٣- نعوض فى معادلة الـ *Once Statically indeterminate* .

$$\delta_{10} + \delta_{11} x X = 0 \longleftarrow \text{Change in Slope angle at point a} = 0$$

$$\delta_{10} = \int \frac{M_1 \, M_0}{EI} dL \qquad \delta_{11} = \int \frac{M_1 \, M_1}{EI} dL$$

Where:

$\delta_{10} \Longrightarrow$ Change in Slope angle at point a
(For the main system)

$\delta_{11} \Longrightarrow$ Change in Slope angle at point a
(For the Correction system)

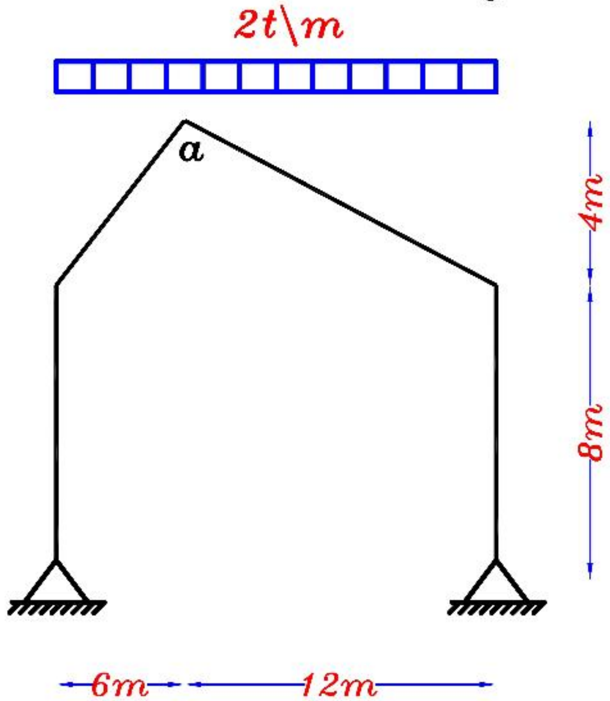
٤- بحل ابمعادلة نحصل على المجهول *X* وهو قيمة الـ *Moment* عند *Point a*

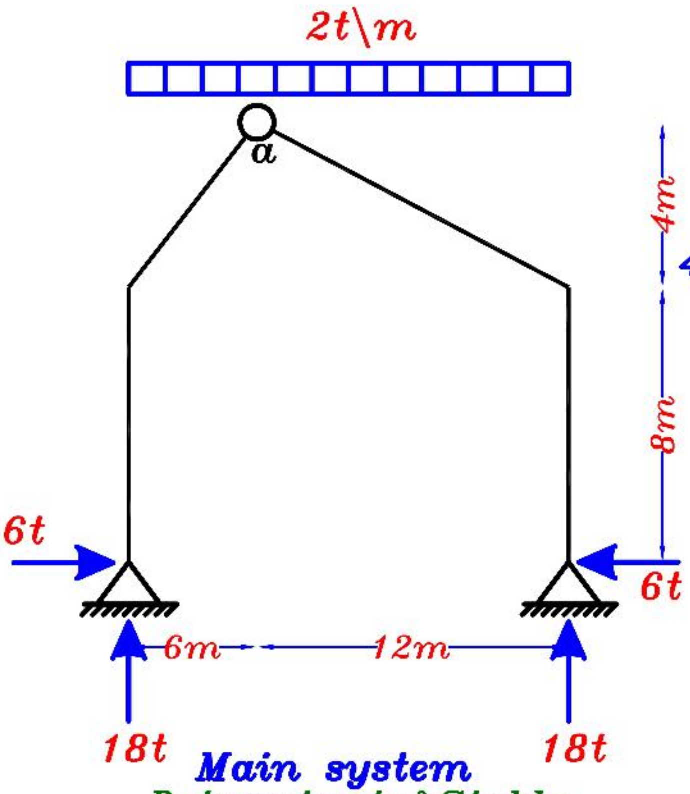
٥- لحساب الـ *Reactions* أو رسم الـ *B.M.D* .

$$\boxed{M_{final} = M_0 + (X) M_1} \qquad \boxed{R_{final} = R_0 + (X) R_1}$$

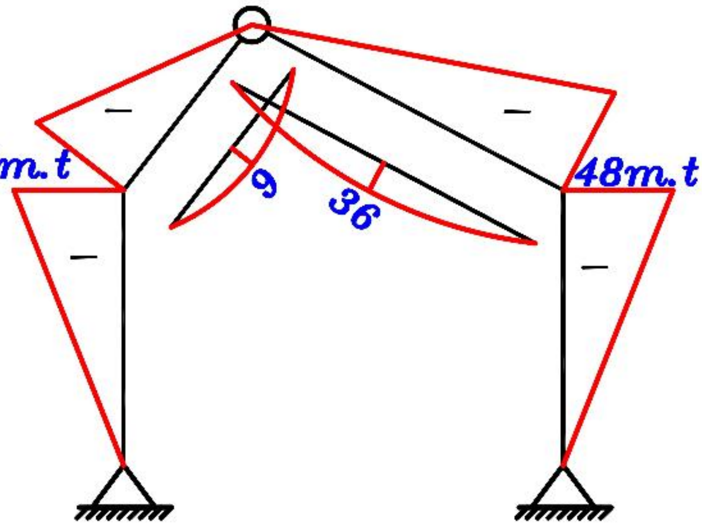
Example:

For the shown frame draw the B.M.D by increasing equations .

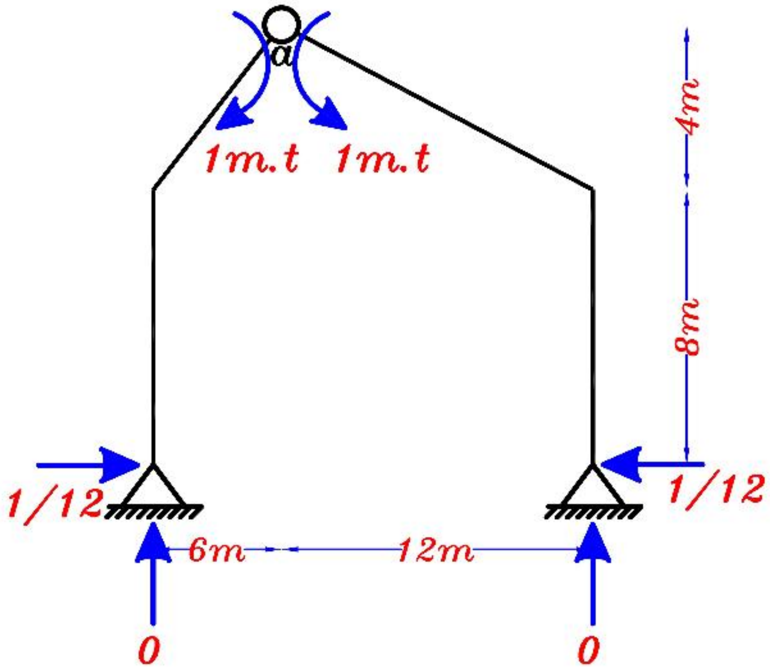




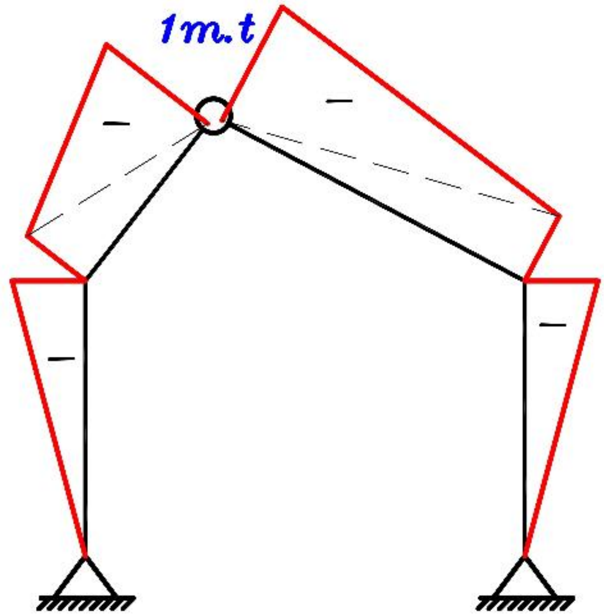
Main system
Determinate&Stable



M_0



Correction system(1)



M_1

$$\delta_{10} + \delta_{11} x X = 0$$

$$\delta_{10} = \int \frac{M_1\, M_0}{EI}\, dL \qquad \delta_{11} = \int \frac{M_1\, M_1}{EI}\, dL$$

$$\delta_{10} = \frac{1}{EI}[(1/2x48x8)(2/3x0.67)x2$$
$$+ (1/2x48x7.21)(2/3x0.67+1/3x1)$$
$$-(2/3x7.21x9)(1/2x0.67+1/2x1)$$
$$+(1/2x48x12.65)(2/3x0.67+1/3x1)$$
$$-(2/3x12.65x36)(1/2x0.67+1/2x1)] = \frac{252.33}{EI}$$

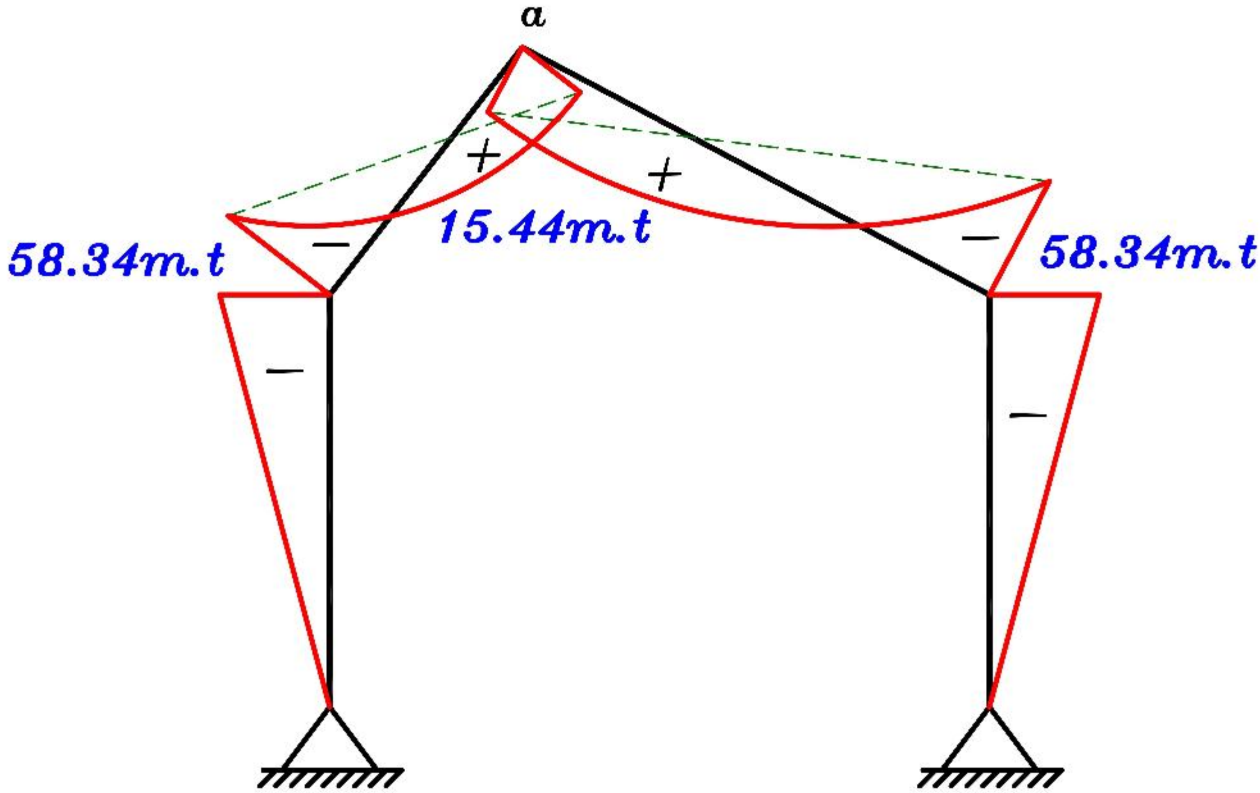
$$\delta_{11} = \frac{1}{EI}[(1/2x0.67x8)(2/3x8)x2$$
$$+(1/2x7.21x0.67)(2/3x0.67+1/3x1)$$
$$+(1/2x7.21x1)(2/3x1+1/3x0.67)$$
$$+(1/2x12.65x0.67)(2/3x0.67+1/3x1)$$
$$+(1/2x12.65x1)\ (2/3x1+1/3x0.67)] = \frac{16.34}{EI}$$

$$\delta_{10} + \delta_{11} x X = 0$$

$$\frac{252.33}{EI} + \frac{16.34}{EI} \; x X = 0 \Longrightarrow \boxed{X = -15.44 m.t}$$

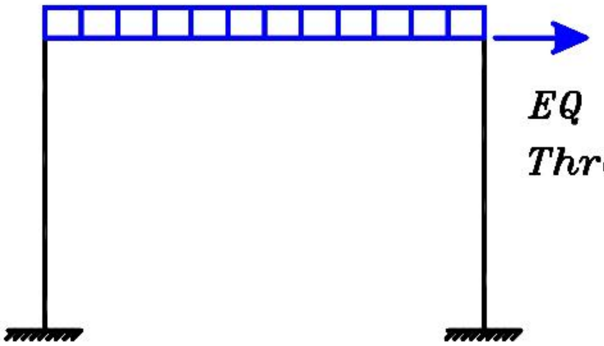
$$\boxed{M_{final} = M_0 + (-15.44) M_1}$$

$$\boxed{R_{final} = R_0 + (-15.44) R_1}$$



Final B.M.D

3) Three times statically indeterminate:



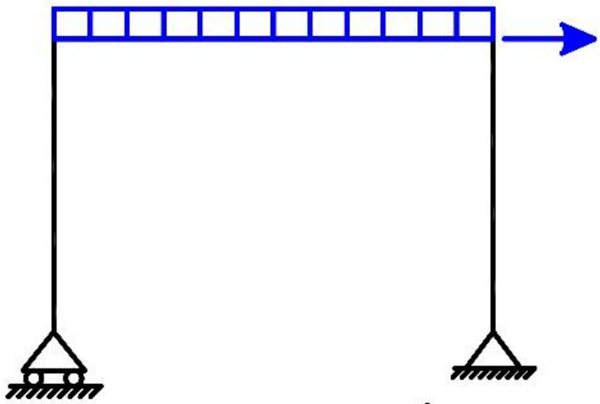
$UN = 6$ & $EQ = 3$

$EQ < UN$ ----- Indeterminate structure

Three times statically indeterminate

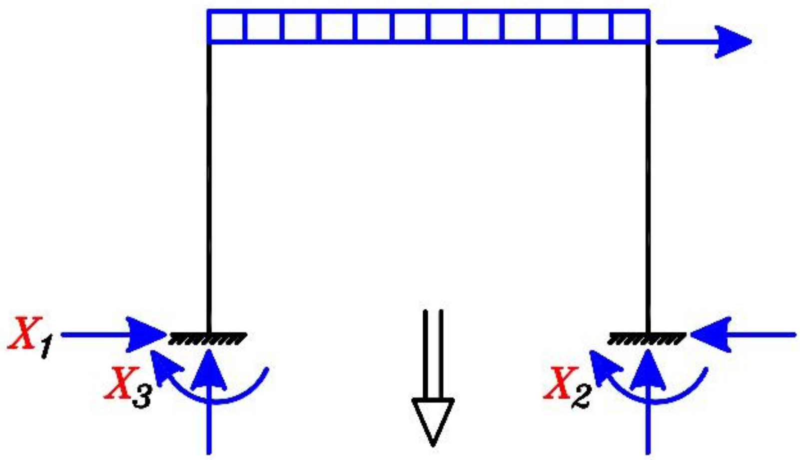
نأخذ المجاهيل هى ال*Moment* ال*Fixed Supports* و ال*Horizontal Reaction*

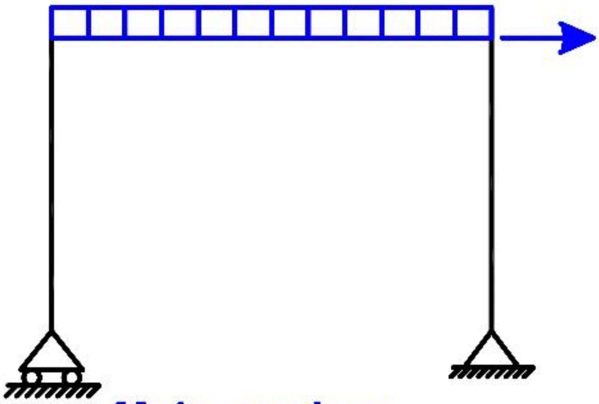
لل*Hinged Support* الشمال .



من الممكن أخذ أى مجاهيل أخرى ولكن بشرط أن يظل ال *System* ⟸ *Stable*

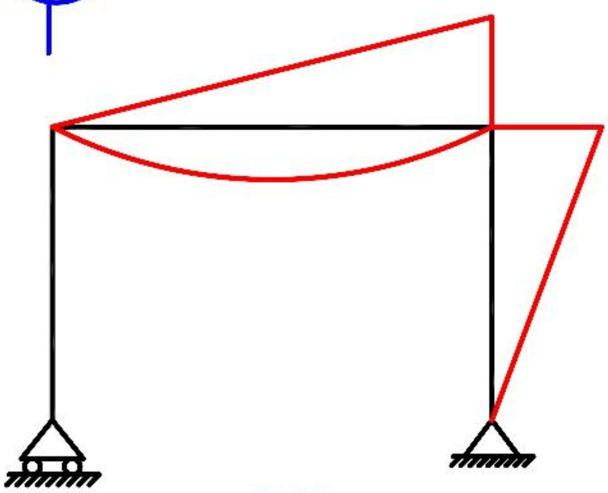
و يصبح *Determinate*.



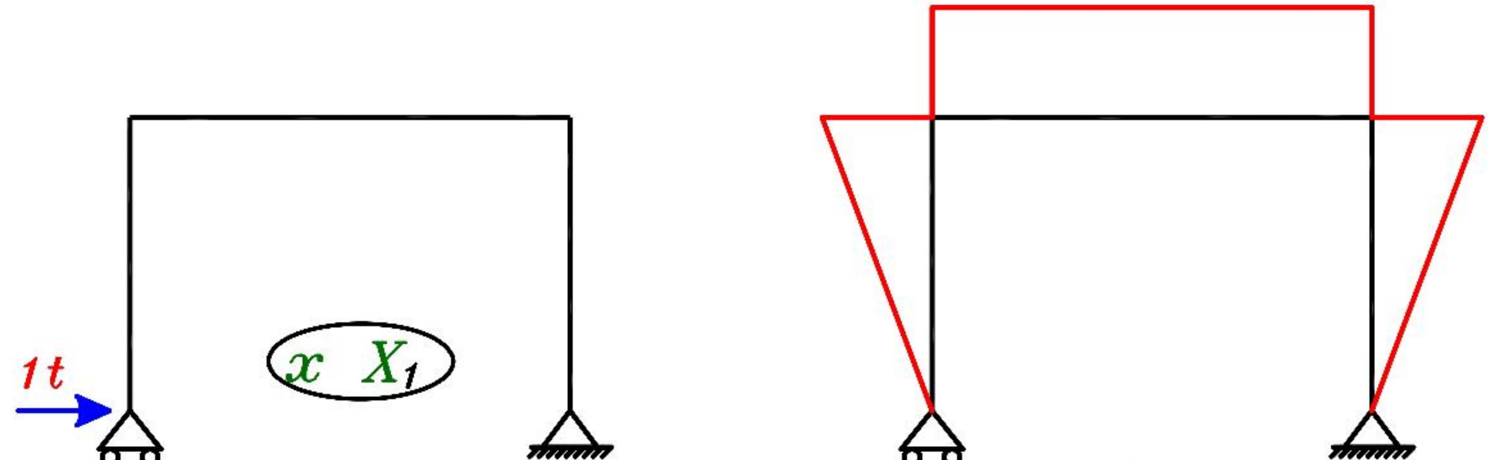


Main system

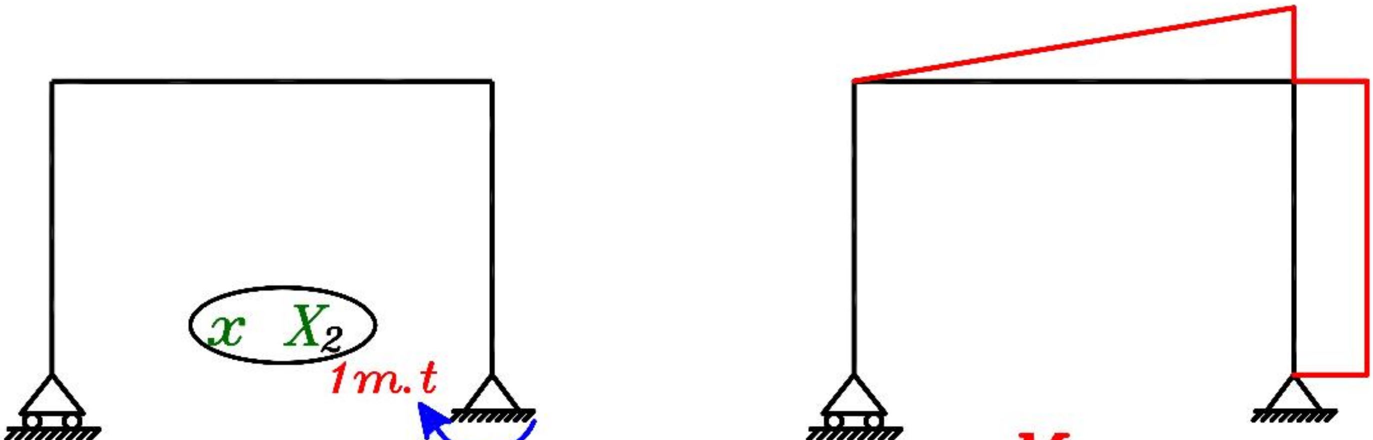
Determinate&Stable



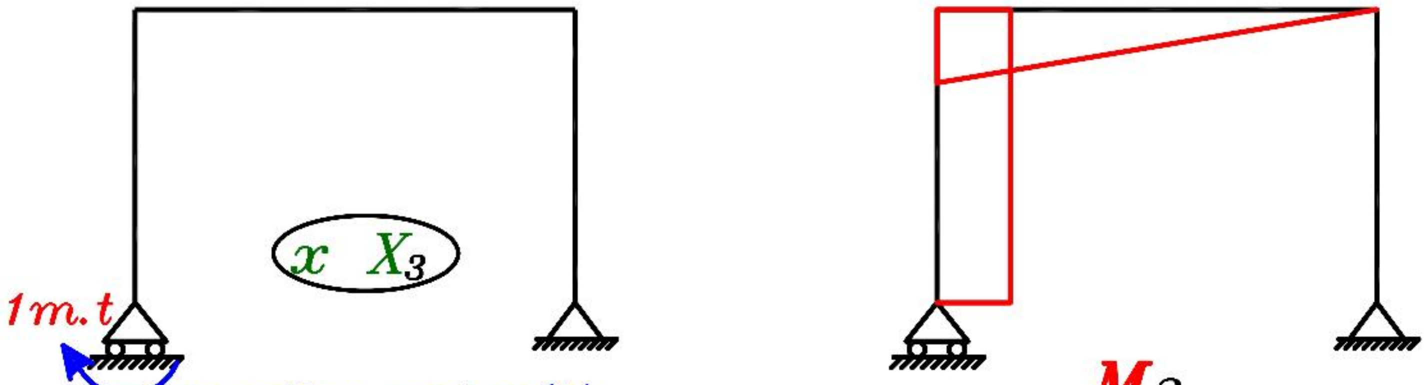
M_0



Correction system(1) $\quad\quad$ **M_1**



Correction system(2) $\quad\quad$ **M_2**



Correction system(3) $\quad\quad$ **M_3**

$$\delta_{10} + \delta_{11} x X_1 + \delta_{12} x X_2 + \delta_{13} x X_3 = 0$$

$$\delta_{20} + \delta_{21} x X_1 + \delta_{22} x X_2 + \delta_{23} x X_3 = 0$$

$$\delta_{30} + \delta_{31} x X_1 + \delta_{32} x X_2 + \delta_{33} x X_3 = 0$$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \qquad \delta_{20} = \int \frac{M_2 M_0}{EI} dL \qquad \delta_{30} = \int \frac{M_3 M_0}{EI} dL$$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL \qquad \delta_{21} = \int \frac{M_2 M_1}{EI} dL \qquad \delta_{31} = \int \frac{M_3 M_1}{EI} dL$$

$$\delta_{12} = \int \frac{M_1 M_2}{EI} dL \qquad \delta_{22} = \int \frac{M_2 M_2}{EI} dL \qquad \delta_{32} = \int \frac{M_3 M_2}{EI} dL$$

$$\delta_{13} = \int \frac{M_1 M_3}{EI} dL \qquad \delta_{23} = \int \frac{M_2 M_3}{EI} dL \qquad \delta_{33} = \int \frac{M_3 M_3}{EI} dL$$

$$\boxed{\begin{array}{c} \delta_{21} = \delta_{12} \qquad \delta_{31} = \delta_{13} \\ \delta_{32} = \delta_{23} \end{array}}$$

$$\boxed{M_{final} = M_0 + (X_1) M_1 + (X_2) M_2 + (X_3) M_3}$$

$$\boxed{R_{final} = R_0 + (X_1) R_1 + (X_2) R_2 + (X_3) R_3}$$

1) Once statically indeterminate:

$$\delta_{10} + \delta_{11} x X = 0$$

$$\boxed{M_{final} = M_0 + (X) M_1} \quad \boxed{R_{final} = R_0 + (X) R_1}$$

2) Twice statically indeterminate:

$$\delta_{10} + \delta_{11} x X_1 + \delta_{12} x X_2 = 0$$

$$\delta_{20} + \delta_{21} x X_1 + \delta_{22} x X_2 = 0$$

$$\boxed{M_{final} = M_0 + (X_1) M_1 + (X_2) M_2}$$

$$\boxed{R_{final} = R_0 + (X_1) R_1 + (X_2) R_2}$$

3) Three times statically indeterminate:

$$\delta_{10} + \delta_{11} x X_1 + \delta_{12} x X_2 + \delta_{13} x X_3 = 0$$

$$\delta_{20} + \delta_{21} x X_1 + \delta_{22} x X_2 + \delta_{23} x X_3 = 0$$

$$\delta_{30} + \delta_{31} x X_1 + \delta_{32} x X_2 + \delta_{33} x X_3 = 0$$

$$\boxed{M_{final} = M_0 + (X_1) M_1 + (X_2) M_2 + (X_3) M_3}$$

$$\boxed{R_{final} = R_0 + (X_1) R_1 + (X_2) R_2 + (X_3) R_3}$$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \qquad \delta_{20} = \int \frac{M_2 M_0}{EI} dL \qquad \delta_{30} = \int \frac{M_3 M_0}{EI} dL$$

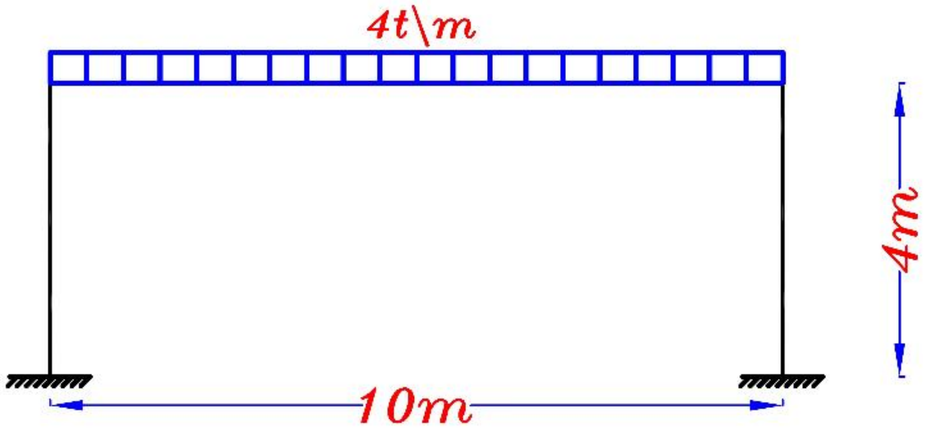
$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL \qquad \delta_{21} = \int \frac{M_2 M_1}{EI} dL \qquad \delta_{31} = \int \frac{M_3 M_1}{EI} dL$$

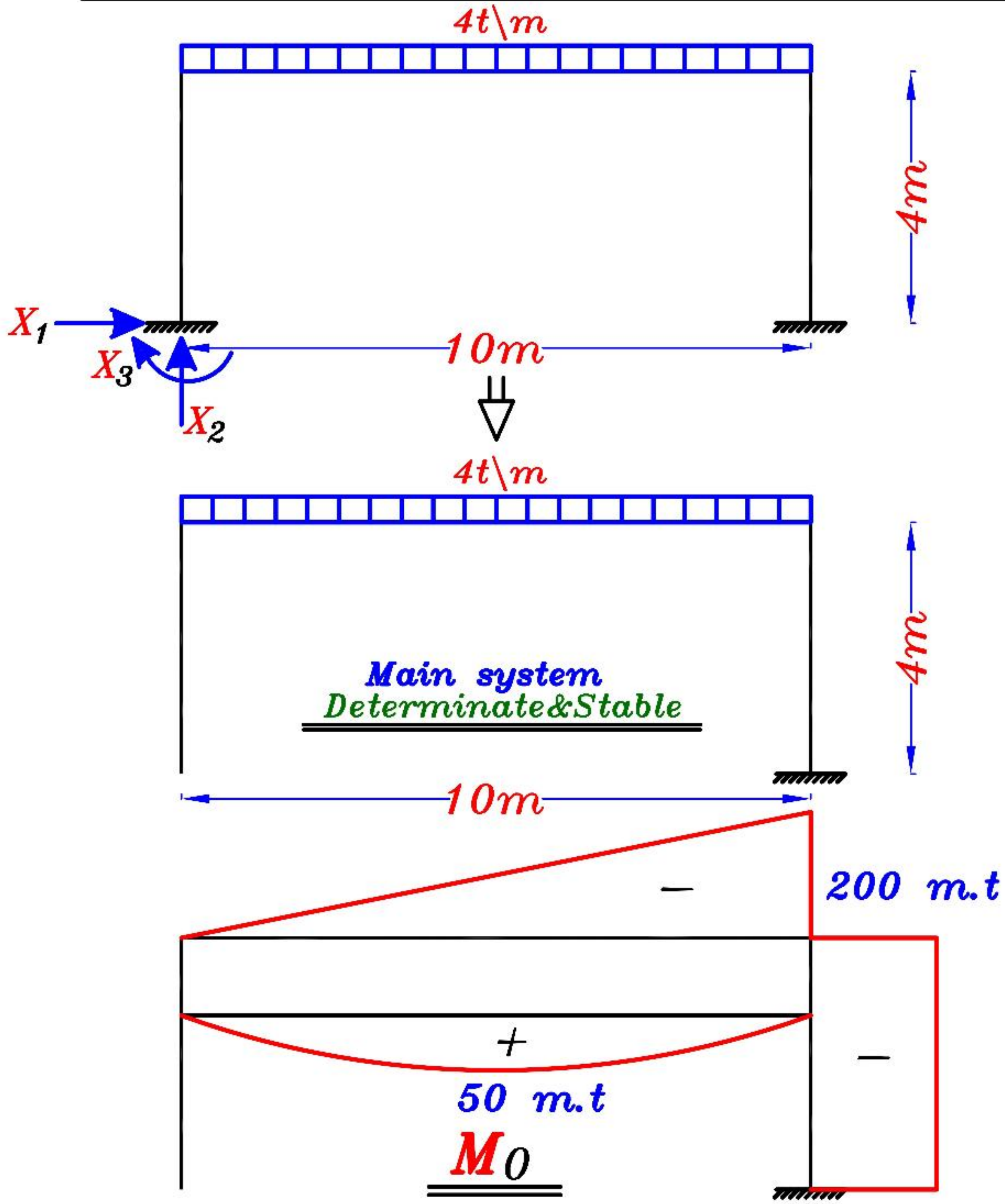
$$\delta_{12} = \int \frac{M_1 M_2}{EI} dL \qquad \delta_{22} = \int \frac{M_2 M_2}{EI} dL \qquad \delta_{32} = \int \frac{M_3 M_2}{EI} dL$$

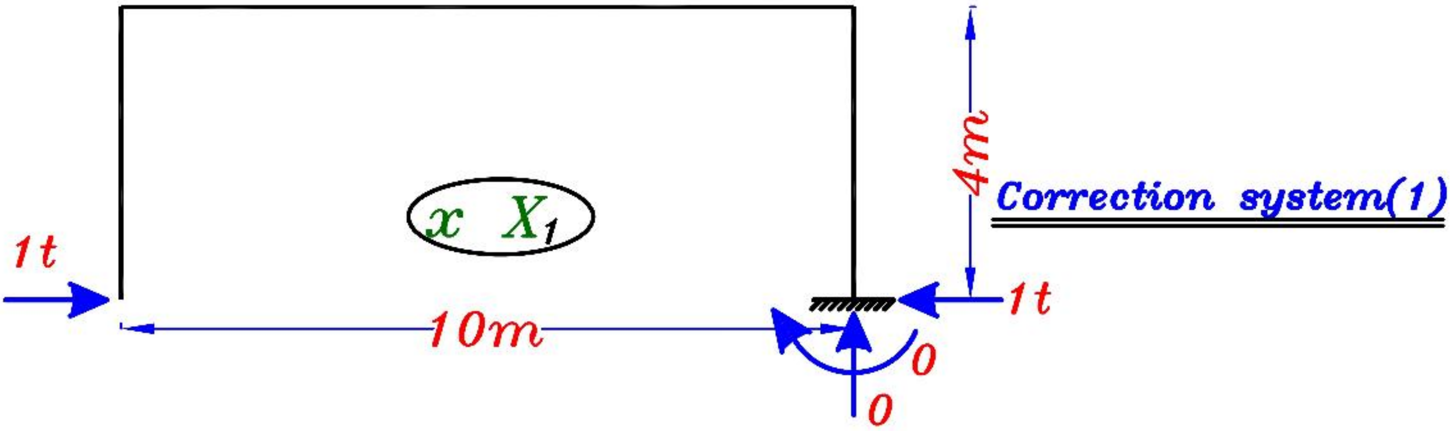
$$\delta_{13} = \int \frac{M_1 M_3}{EI} dL \qquad \delta_{23} = \int \frac{M_2 M_3}{EI} dL \qquad \delta_{33} = \int \frac{M_3 M_3}{EI} dL$$

Example:

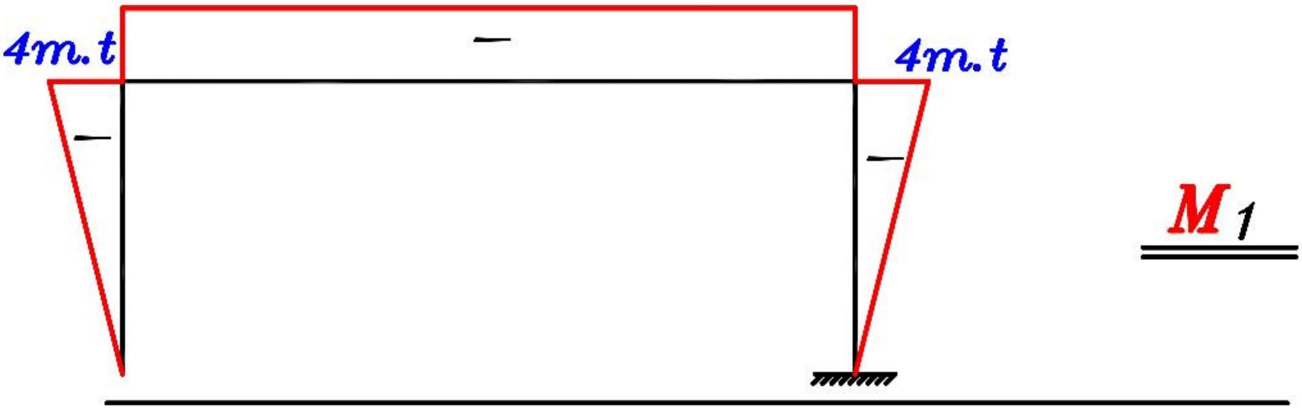
For the shown frame draw the B.M.D .



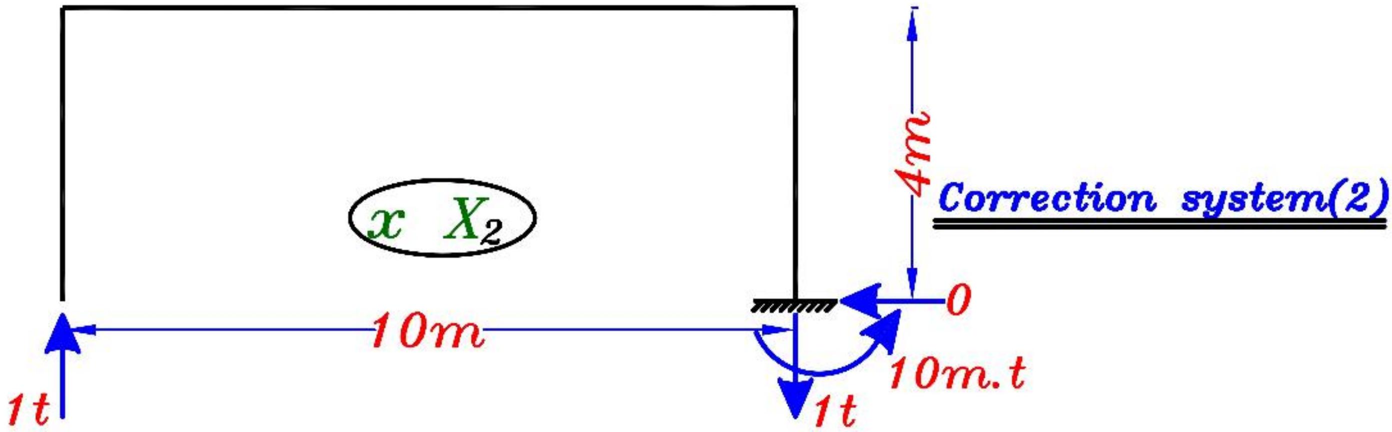




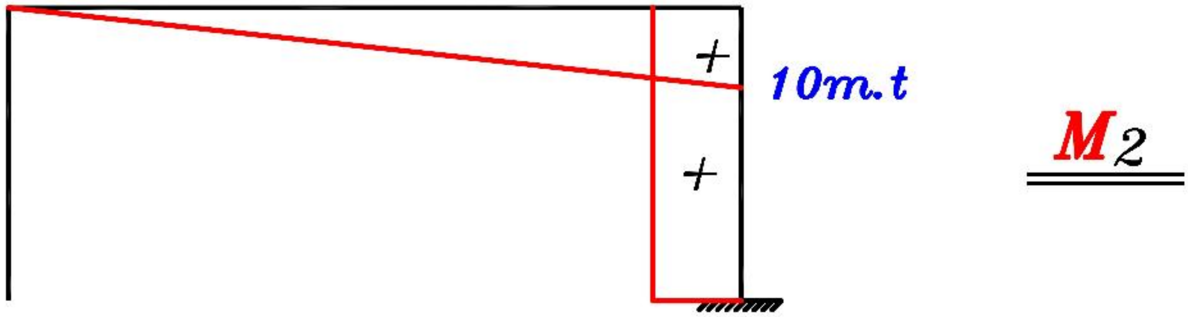
4m
Correction system(1)
x X1
1t
1t
10m
0
0



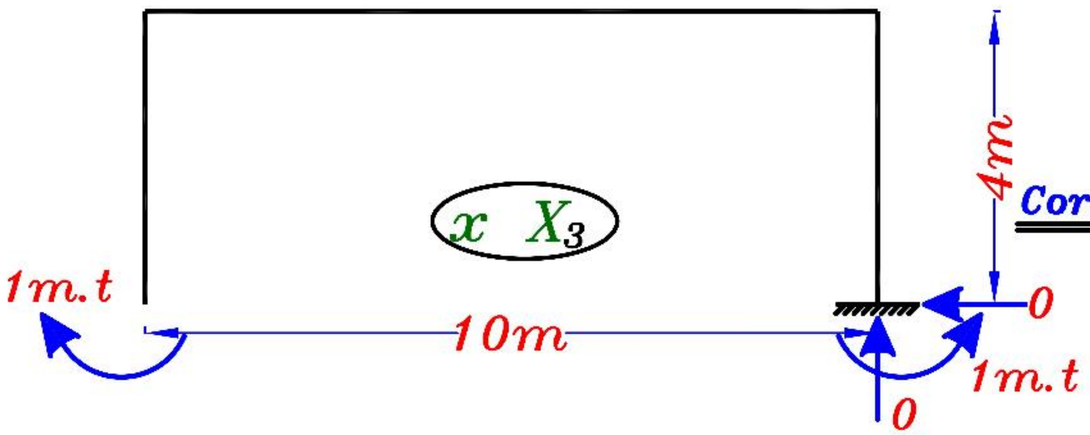
4m.t
4m.t
M1



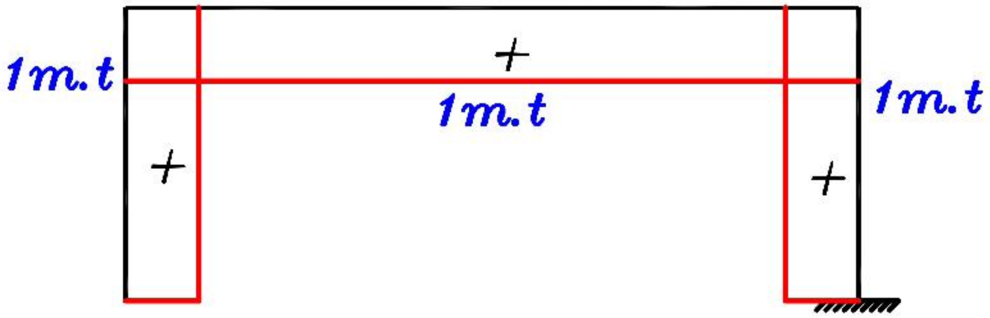
4m
Correction system(2)
x X2
0
10m
10m.t
1t
1t



10m.t
M2



Correction system(3)



M_3

$$\delta_{10} + \delta_{11} \times X_1 + \delta_{12} \times X_2 + \delta_{13} \times X_3 = 0$$
$$\delta_{20} + \delta_{21} \times X_1 + \delta_{22} \times X_2 + \delta_{23} \times X_3 = 0$$
$$\delta_{30} + \delta_{31} \times X_1 + \delta_{32} \times X_2 + \delta_{33} \times X_3 = 0$$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL \qquad \delta_{20} = \int \frac{M_2 M_0}{EI} dL \qquad \delta_{30} = \int \frac{M_3 M_0}{EI} dL$$
$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL \qquad \delta_{21} = \int \frac{M_2 M_1}{EI} dL \qquad \delta_{31} = \int \frac{M_3 M_1}{EI} dL$$
$$\delta_{12} = \int \frac{M_1 M_2}{EI} dL \qquad \delta_{22} = \int \frac{M_2 M_2}{EI} dL \qquad \delta_{32} = \int \frac{M_3 M_2}{EI} dL$$
$$\delta_{13} = \int \frac{M_1 M_3}{EI} dL \qquad \delta_{23} = \int \frac{M_2 M_3}{EI} dL \qquad \delta_{33} = \int \frac{M_3 M_3}{EI} dL$$

$$\delta_{10} = \frac{4266.67}{EI} \qquad \delta_{20} = \frac{-13000}{EI} \qquad \delta_{30} = \frac{1466.67}{EI}$$
$$\delta_{11} = \frac{202.67}{EI} \qquad \delta_{22} = \frac{733.33}{EI} \qquad \delta_{33} = \frac{18}{EI}$$
$$\delta_{12} = \frac{-280}{EI} = \delta_{21} \qquad \delta_{13} = \frac{-56}{EI} = \delta_{31}$$

$$\delta_{23} = \frac{90}{EI} = \delta_{32}$$

$$\frac{4266.67}{EI} + \frac{202.67}{EI} x X_1 + \frac{-280}{EI} x X_2 + \frac{-56}{EI} x X_3 = 0 \Rightarrow Eq.1$$

$$\frac{-13000}{EI} + \frac{-280}{EI} x X_1 + \frac{733.33}{EI} x X_2 + \frac{90}{EI} x X_3 = 0 \Rightarrow Eq.2$$

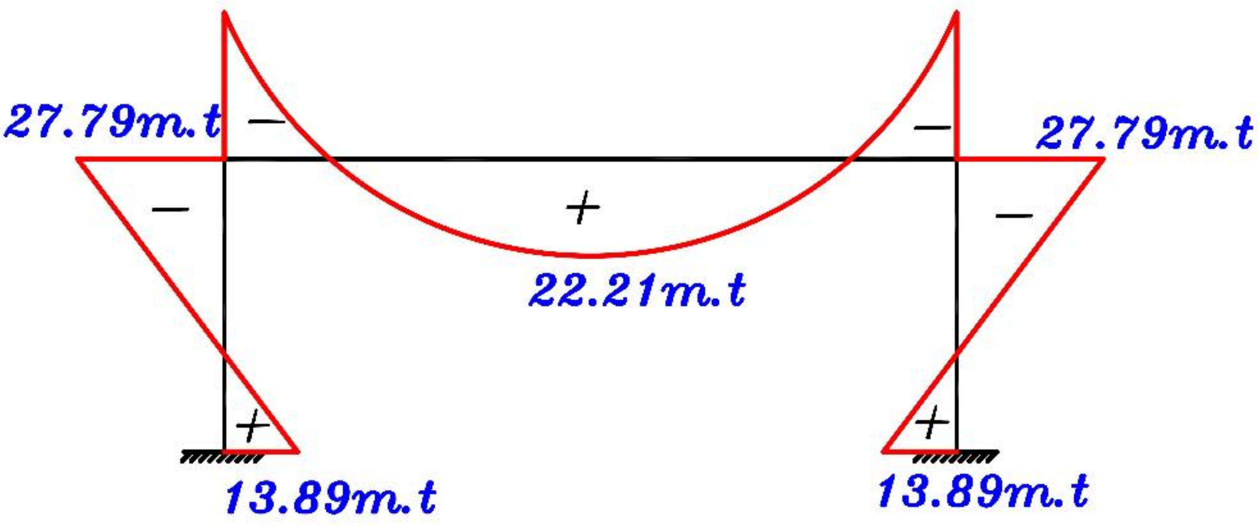
$$\frac{1466.67}{EI} + \frac{-56}{EI} x X_1 + \frac{90}{EI} x X_2 + \frac{18}{EI} x X_3 = 0 \Rightarrow Eq.3$$

Solving the equations :

$X_1 = 10.42t$ $\quad$ $X_2 = 20t$ $\quad$ $X_3 = 13.89m.t$

$$\boxed{M_{final} = M_0 + (10.42)M_1 + (20)M_2 + (13.89)M_3}$$

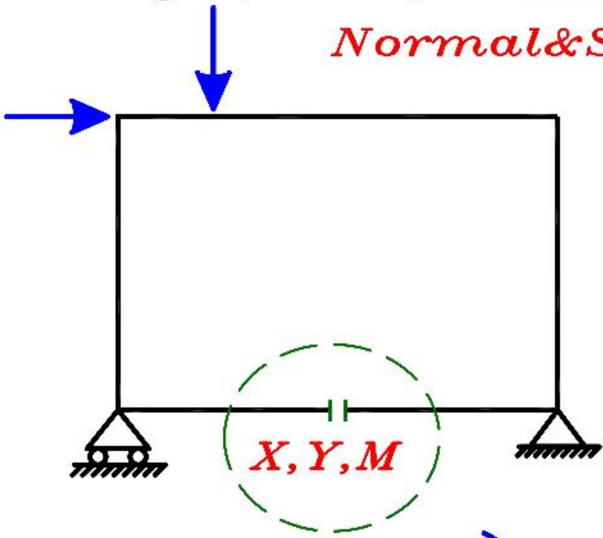
$$\boxed{R_{final} = R_0 + (10.42)R_1 + (20)R_2 + (13.89)R_3}$$

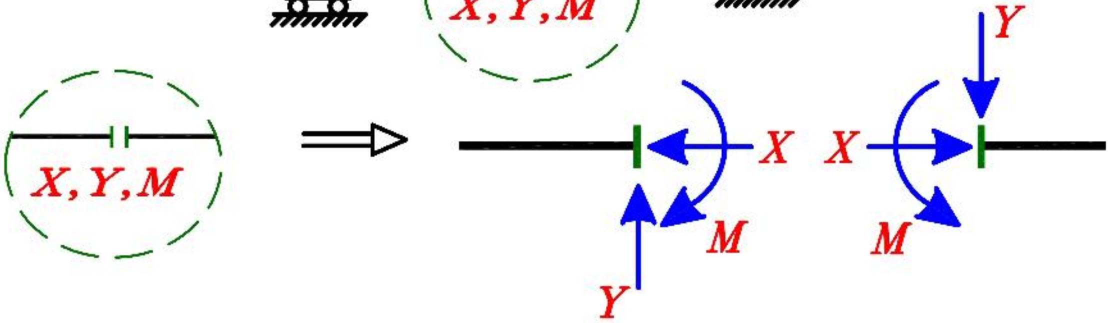


Final B.M.D

Closed Frame

اذا قطعنا الـ Closed Frame عند أى نقطة نجد أن هناك ٣ مجاهيل و هم Normal&Shear and moment

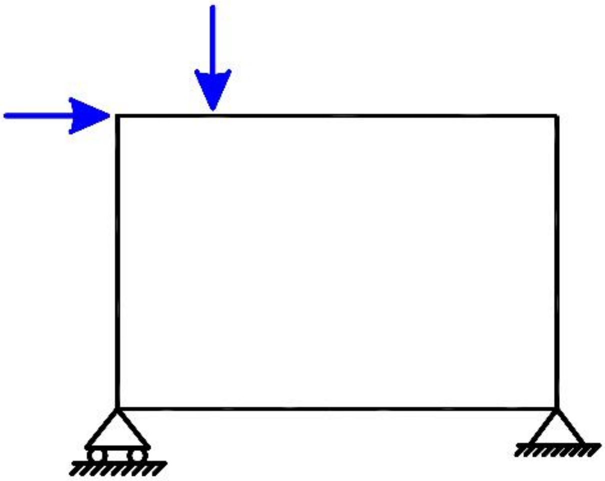




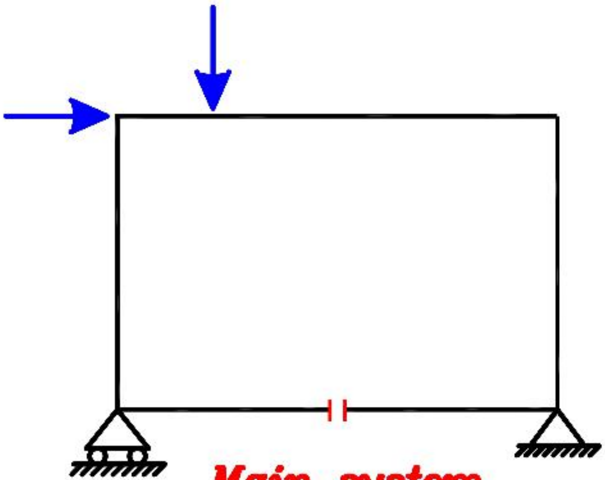
و لحله نقطعه عند أى نقطة و تكون المجاهيل هى الـ X,Y,M بدلا من الـ Reactions .

النقطة التى قطعنا عندها تكون الـ Boundary Conditions لها هى أن الـ Slope angle من اليمين يساوى الـ Slope angle من الشمال و كذلك الـ Horizontal displacement و الـ Vertical disp. و هذا معناه أن الـ Change in Slope angle = 0 و كذلك الـ Change in Horizontal displacement = 0 و كذلك الـ Change in Vertical displacement = 0

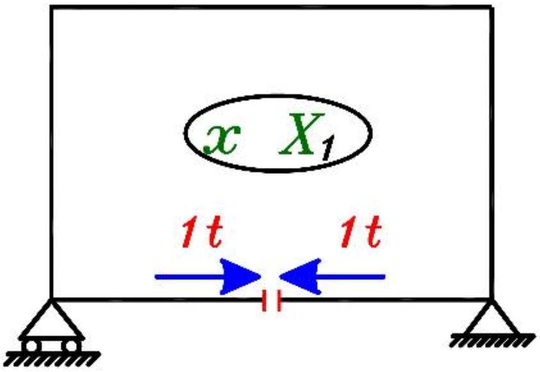
لتحويل الـ Closed Frame الى Determinate Frame نقطع الـ عند أى نقطة و نزيل الـ X,Y,M ثم نضع فى الـ Correction System الاول الـ X مثلا و الثانى الـ Y و الثالث الـ M .



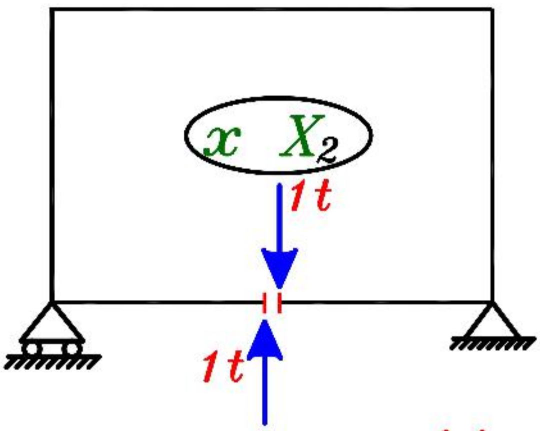
Indeterminate Structure



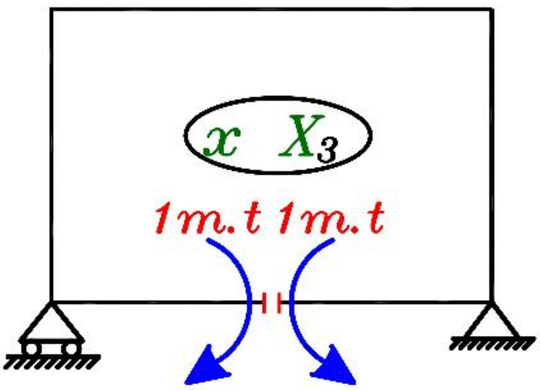
Main system
Determinate&Stable



Correction system(1)



Correction system(2)



Correction system(3)

Change in horizontal dislacement

$$\delta_{10} + \delta_{11} x X_1 + \delta_{12}\ x X_2 + \delta_{13}\ x X_3 = 0$$

Change in vertical dislacement

$$\delta_{20} + \delta_{21} x X_1 + \delta_{22}\ x X_2 + \delta_{23}\ x X_3 = 0$$

Change in Slope angle

$$\delta_{30} + \delta_{31} x X_1 + \delta_{32}\ x X_2 + \delta_{33}\ x X_3 = 0$$

و بحل المعادلات نحصل على المجاهيل $X_1 \& X_2 \& X_3$ حيث

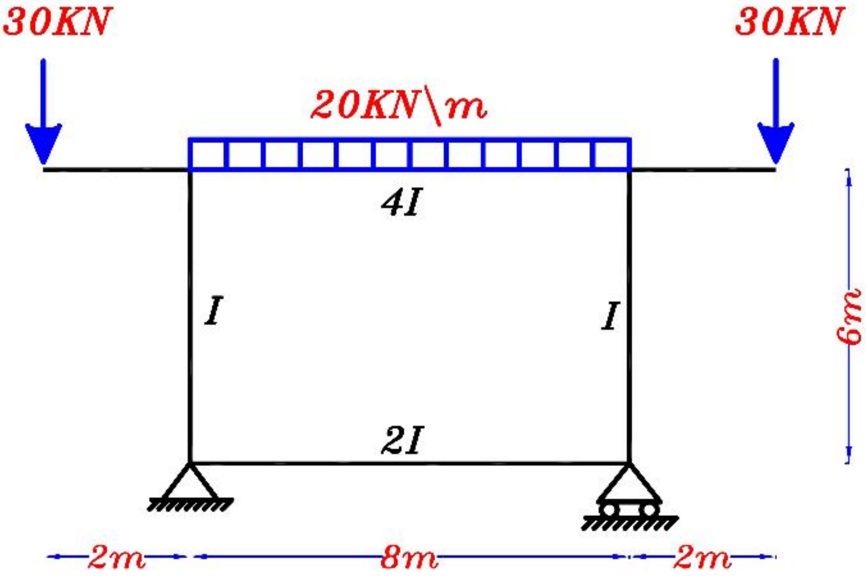
$X_1 = X$ $X_2 = Y$ $X_3 = M$

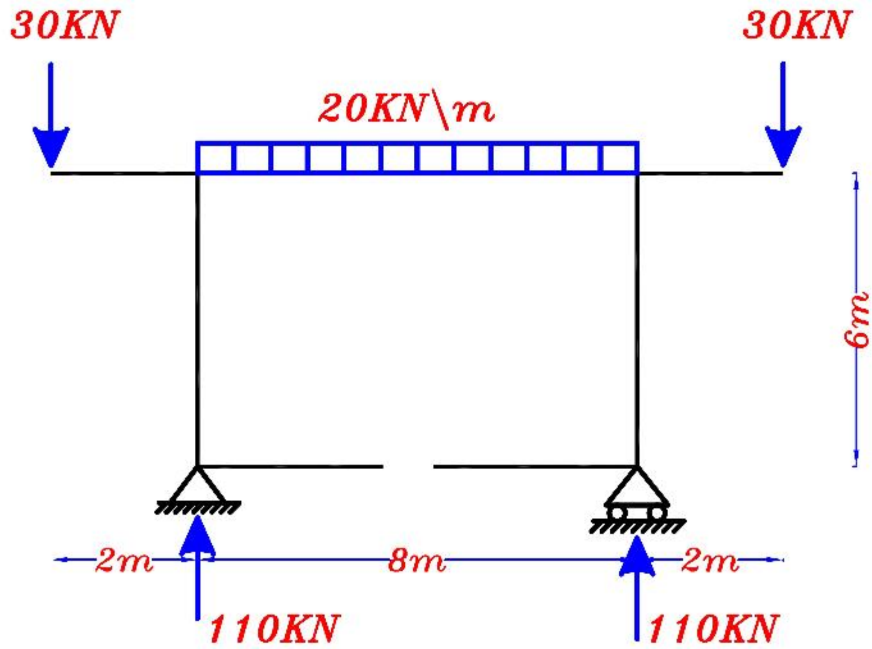
$$\boxed{M_{final} = M_0 + (X_1) M_1 + (X_2) M_2 + (X_3) M_3}$$

$$\boxed{R_{final} = R_0 + (X_1) R_1 + (X_2) R_2 + (X_3) R_3}$$

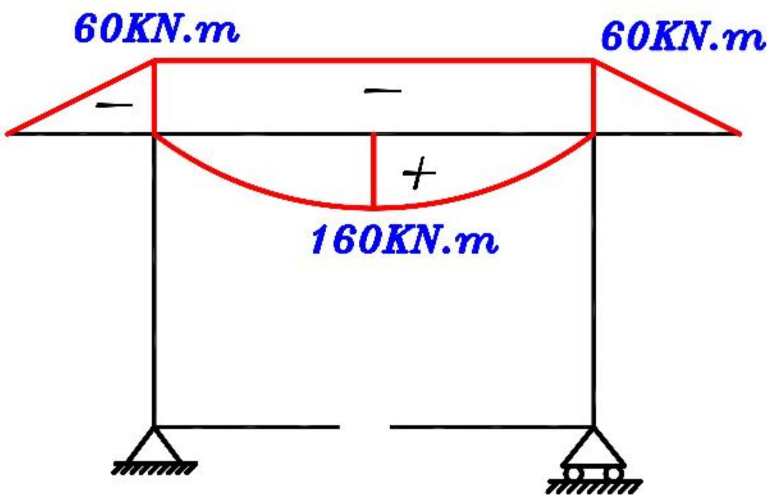
Example:

For the shown frame draw the B.M.D .

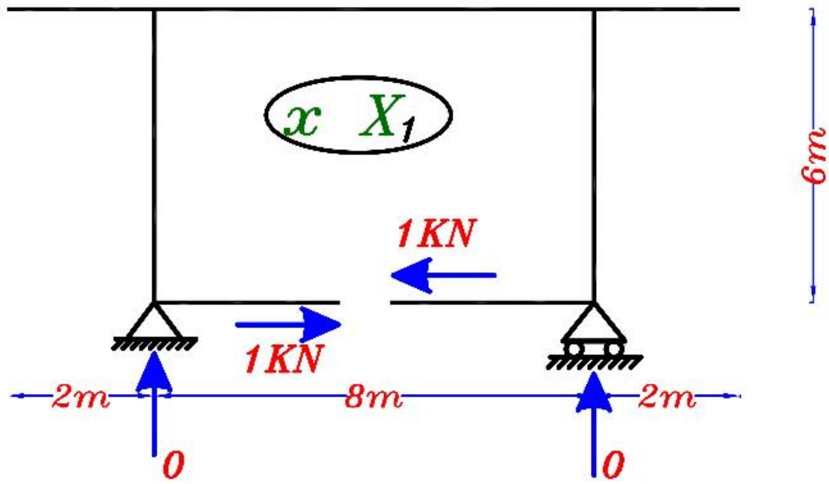




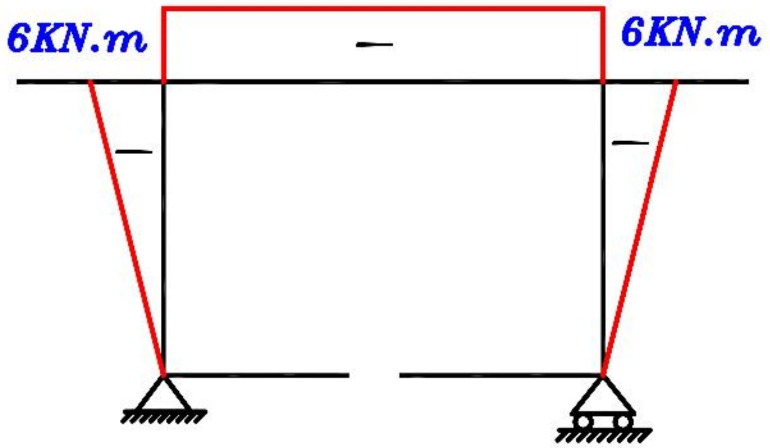
Main system
Determinate&Stable



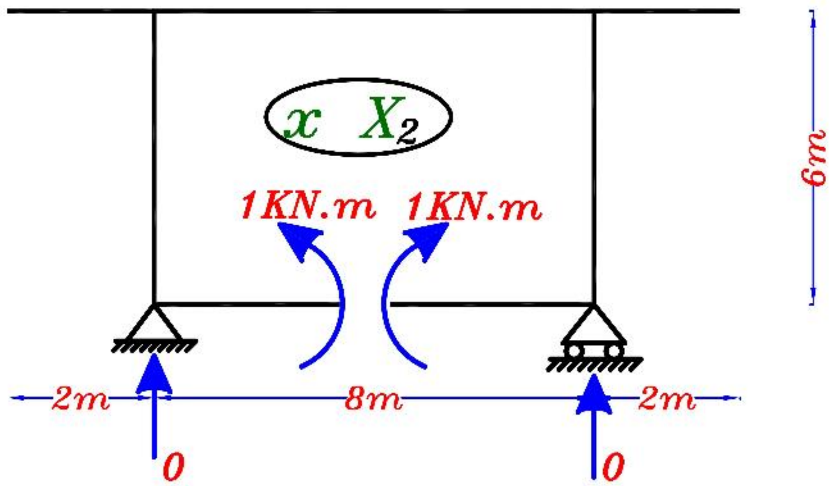
M_0



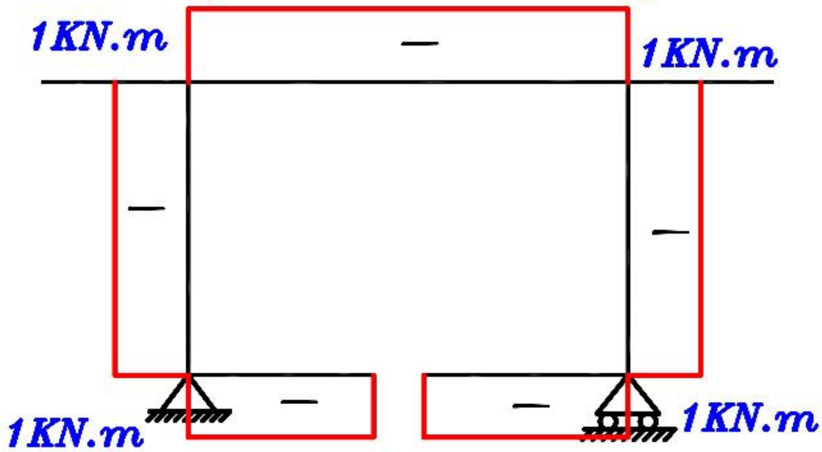
Correction system(1)



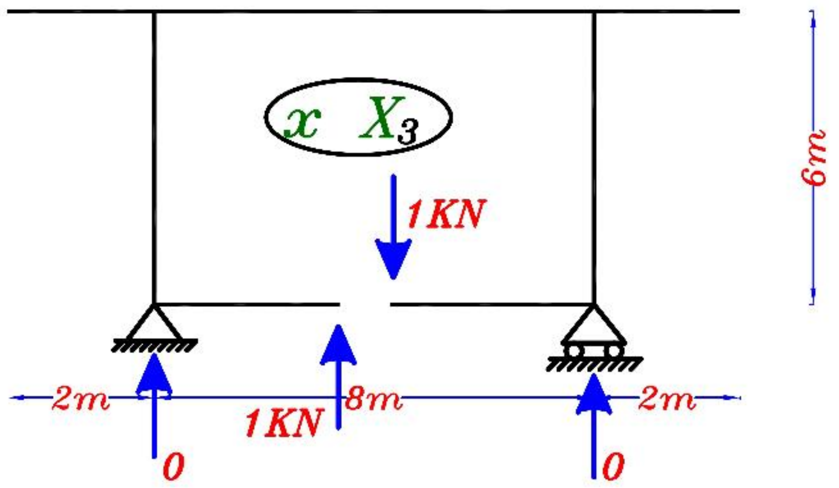
M_1



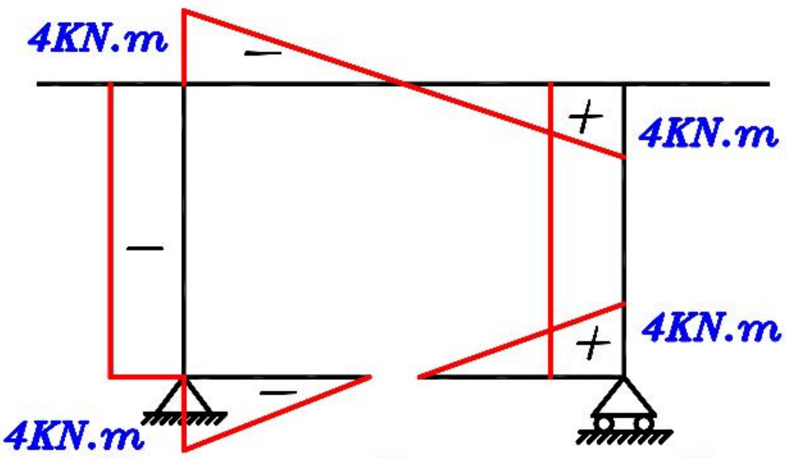
Correction system(2)



M_2



Correction system(3)



M_3

$$\delta_{10} + \delta_{11} x X_1 + \delta_{12} x X_2 + \delta_{13} x X_3 = 0$$
$$\delta_{20} + \delta_{21} x X_1 + \delta_{22} x X_2 + \delta_{23} x X_3 = 0$$
$$\delta_{30} + \delta_{31} x X_1 + \delta_{32} x X_2 + \delta_{33} x X_3 = 0$$

$$\delta_{10} = \int \frac{M_1 M_0}{EI} dL$$

$$\delta_{10} = \frac{1}{4EI}[(60x8)(6) - (2/3x160x8)(6)] = \frac{-560}{EI}$$

$$\delta_{20} = \int \frac{M_2 M_0}{EI} dL$$

$$\delta_{20} = \frac{1}{4EI}[(60x8)(1) - (2/3x160x8)(1)] = \frac{-93.33}{EI}$$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$\delta_{11} = \frac{1}{EI}[(1/2x6x6)(2/3x6)x2] + \frac{1}{4EI}[(6x8)(6)] = \frac{216}{EI}$$

$$\delta_{22} = \int \frac{M_2\, M_2}{EI}\, dL$$

$$\delta_{22} = \frac{1}{EI}\left[(1x6)(1)x2\right] + \frac{1}{2EI}\left[(1x8x1)(6)\right] + \frac{1}{4EI}\left[(1x8)(6)\right] = \frac{12}{EI}$$

$$\delta_{12} = \int \frac{M_1\, M_2}{EI}\, dL$$

$$\delta_{12} = \frac{1}{EI}\left[(1/2x6x6)(1)x2\right] + \frac{1}{4EI}\left[(6x8)(1)\right] = \frac{48}{EI} = \delta_{21}$$

$$\delta_{33} = \int \frac{M_3\, M_3}{EI}\, dL$$

$$\delta_{33} = \frac{1}{4EI}\left[(1/2x4x4)(2/3x4)x2\right] + \frac{1}{2EI}\left[(1/2x4x4)(2/3x4)x2\right] \frac{1}{EI}\left[(4x6)(4)x2\right] = \frac{213.33}{EI}$$

$$\delta_{23} = \delta_{32} = \delta_{13} = \delta_{31} = \delta_{30} = 0$$

$$-560 + 216\; x\, X_1 + 48\; x\, X_2 + 0\; x\, X_3 = 0$$

$$-93.33 + 48\; x\, X_1 + 12\; x\, X_2 + 0\; x\, X_3 = 0$$

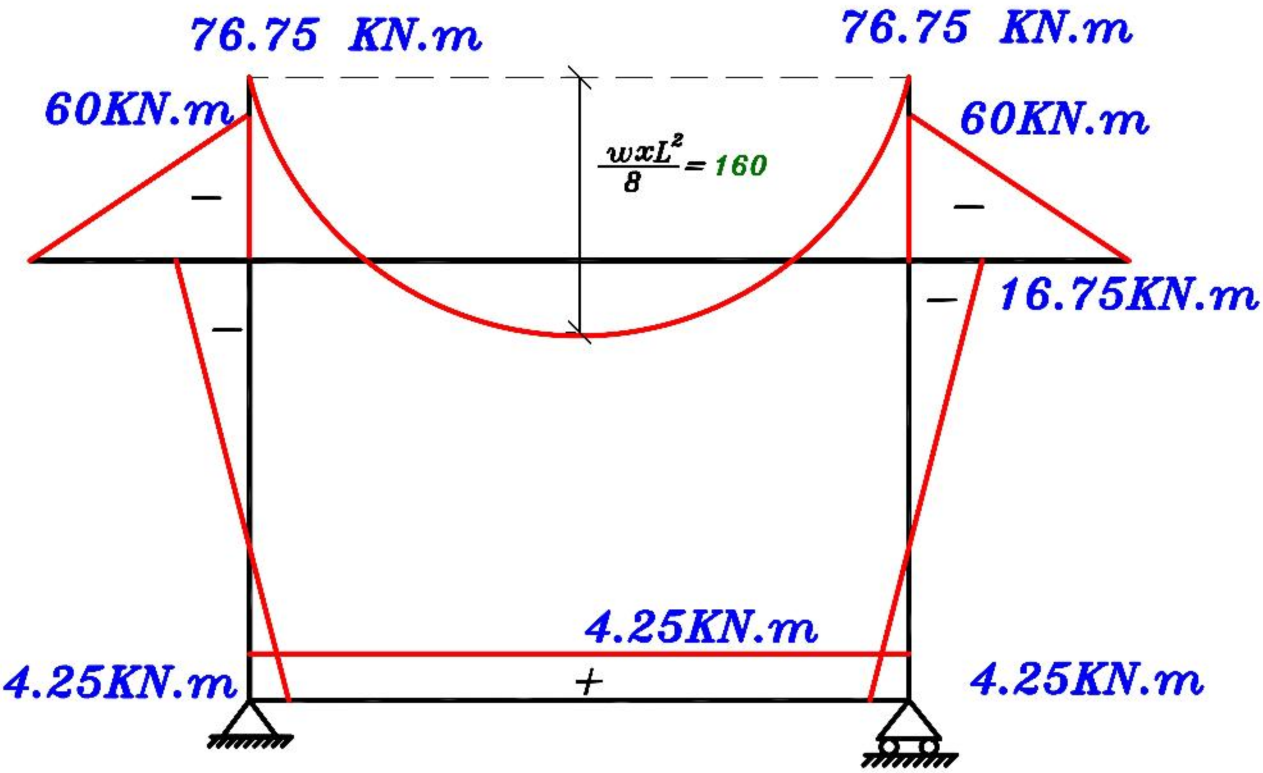
$$0 + 0\; x\, X_1 + 0\; x\, X_2 + 213.33\, x\, X_3 = 0$$

Solving the equations :

$X_1 = 3.5t$ $\quad$ $X_2 = -4.25t$ $\quad$ $X_3 = 0$

$$\boxed{M_{final} = M_0 + (3.5)M_1 + (-4.25)M_2 + (0)M_3}$$

$$\boxed{R_{final} = R_0 + (3.5)R_1 + (-4.25)R_2 + (0)R_3}$$

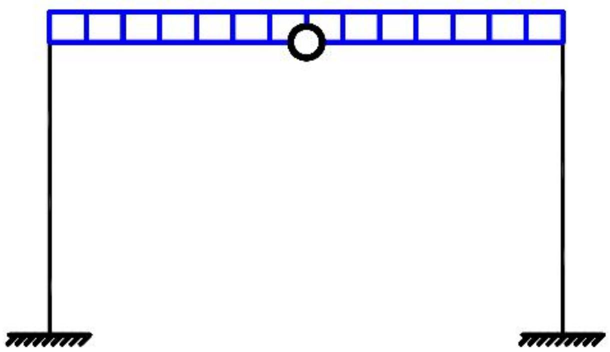

76.75 KN.m
76.75 KN.m
60KN.m
60KN.m
$\frac{wxL^2}{8} = 160$
16.75KN.m
4.25KN.m
4.25KN.m
4.25KN.m

Final B.M.D

Symmetry

فى حالة وجود *Indeterminate Structure* متماثل يمكننا من تقليل عدد ال *Correction Systems* .

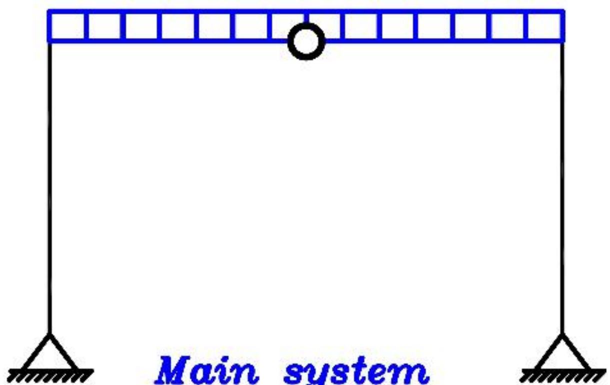
Example:



$UN = 6$ & $EQ = 4$ $EQ < UN$ ----- *Indet. structure*

$UN - EQ = 6 - 4 = 2$ *Twice statically indetermiante*

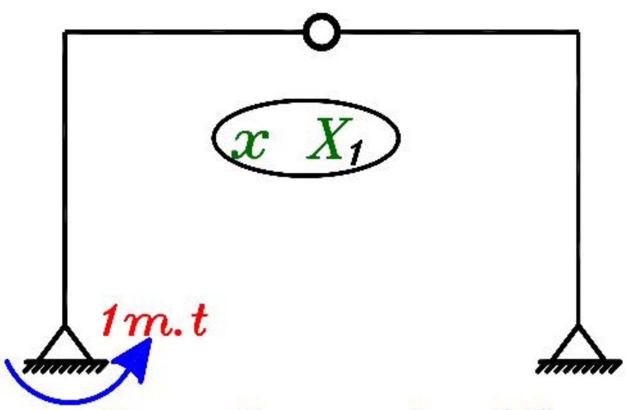
و الحل الطبيعى أن نكون ال *Main system* كالتالى مثلا



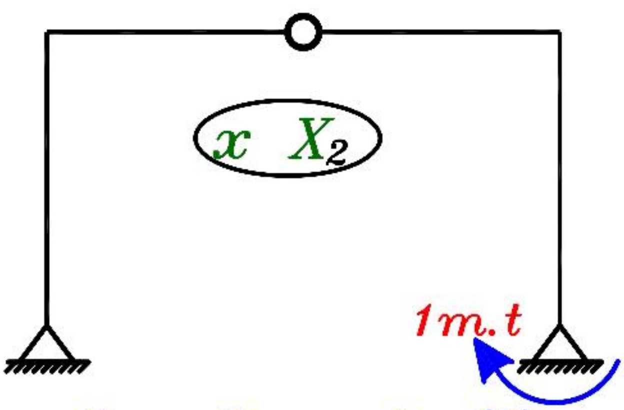
Main system

Determinate&Stable

ثم نكون ال *Correction system* الاول و الثانى



Correction system(1)



Correction system(2)

و لان الـ *Frame* عبارة عن *Twice statically indetermiante* سوف تكون معادلات الحل كالتالى

Twice statically indeterminate:

$$\delta_{10} + \delta_{11} x X_1 + \delta_{12} x X_2 = 0$$
$$\delta_{20} + \delta_{21} x X_1 + \delta_{22} x X_2 = 0$$

$$\boxed{M_{final} = M_0 + (X_1) M_1 + (X_2) M_2}$$

$$\boxed{R_{final} = R_0 + (X_1) R_1 + (X_2) R_2}$$

أى أننا نحتاج لحساب ستة قيم للـ δ و لانه توجد قيمتين متساويتين $\delta_{21} = \delta_{12}$ فسوف نحتاج لحساب خمسة قيم للـ δ .

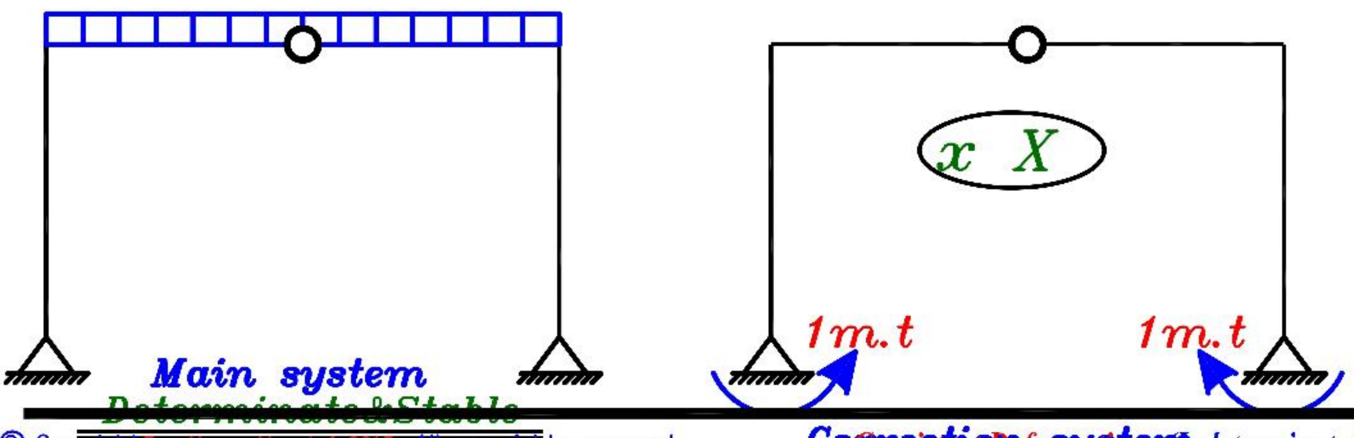
$X_1 = X_2$ لان الـ *Frame* متماثل .

الخلاصة

لحل هذه المسألة نحتاج الى رسم ثلاثة *B.M.D* و حساب خمسة قيم للـ δ .

فى حالة الاستفادة من الـ *Symmetry*

بدلا من عمل *two Correction systems* من الممكن عمل *Correction system* واحد و نضع فيه مجهول واحد و هو الـ X لان $X_1 = X_2$



و بالتالى يكون المجهول الوحيد هو الـ X أى أن المسألة تصبح *Once indet.*
و تصبح المعادلات كالتالى

Once statically indeterminate:

$$\delta_{10} + \delta_{11} x X = 0$$

$$\boxed{M_{final} = M_0 + (X) M_1} \quad \boxed{R_{final} = R_0 + (X) R_1}$$

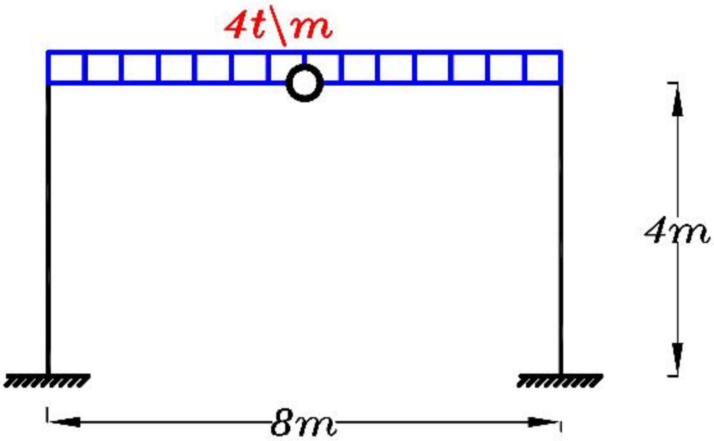
و بالتالى سوف نحتاج الى حساب قيمتين فقط للـ δ .

الخلاصة

لحل هذه المسألة نحتاج الى رسم اثنين *B.M.D* و حساب قيمتين للـ δ .

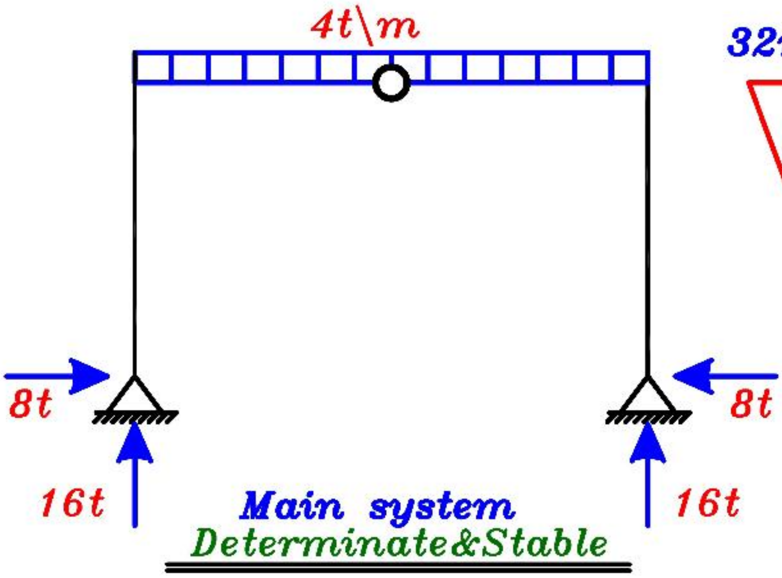
Example:

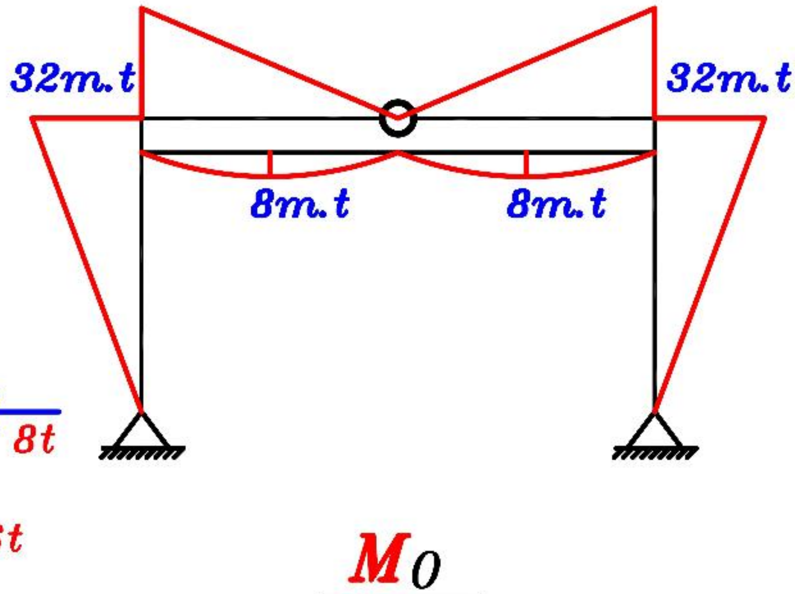
For the shown frame draw the B.M.D .

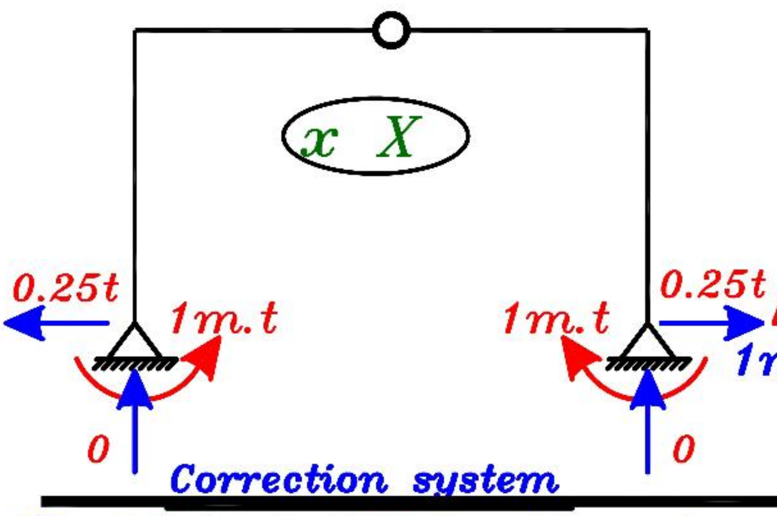


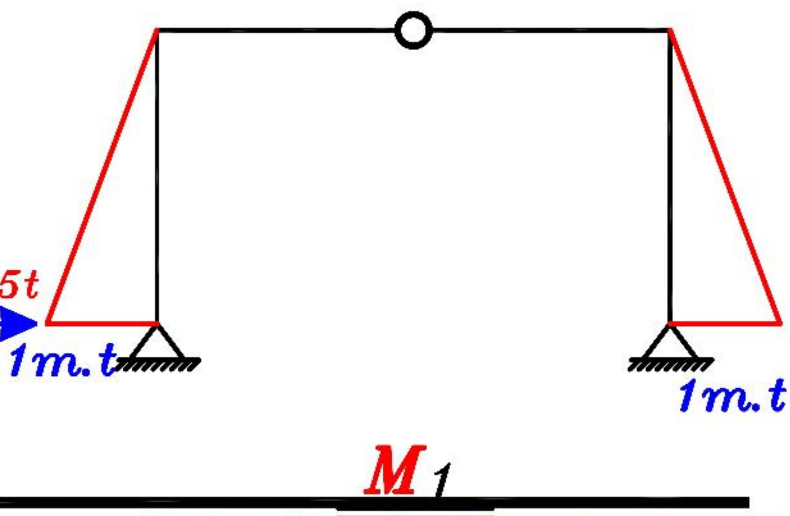
UN = 6 & EQ = 4 EQ < UN ----- Indet. structure

UN − EQ = 6 − 4 = 2 Twice statically indetermiante









$$\delta_{10} + \delta_{11} x\, X = 0$$

$$\delta_{10} = \int \frac{M_1\, M_0}{EI}\, dL \qquad \delta_{11} = \int \frac{M_1\, M_1}{EI}\, dL$$

$$\delta_{10} = \frac{1}{EI}\left[(1/2x32x4)(1/3x1)x2\right] = \frac{42.67}{EI}$$

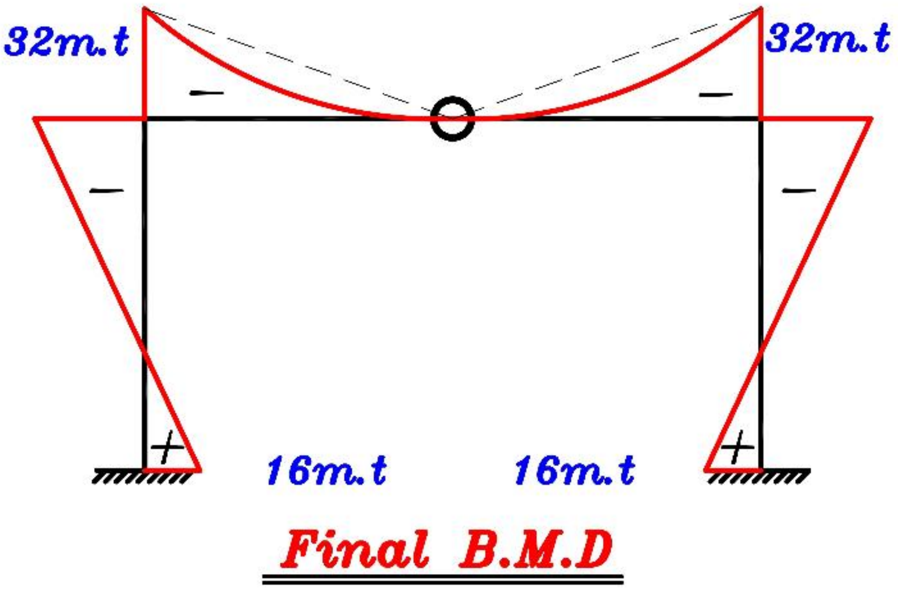
$$\delta_{11} = \frac{1}{EI}\left[(1/2x4x1)\ (2/3x1)x2\right] = \frac{2.67}{EI}$$

$$\delta_{10} + \delta_{11} x\, X = 0$$

$$\frac{42.67}{EI} + \frac{2.67}{EI} \quad x\, X = 0 \Longrightarrow X = -16m.t$$

$$\boxed{M_{final} = M_0 + (-16)M_1}$$

$$\boxed{R_{final} = R_0 + (-16)R_1}$$



Final B.M.D